06B: Linear Shrinkage Strain to Volumetric Strain

In this problem you are asked to relate the change in the reduction in the pore radius to the ***linear*** shrinkage.

There are two ways to address this problem. Please try both of them to see that you get the same answer

In this problem you are asked to draw a relationship between the change in the radius of the pores and Relationships between Linear and Volumetric Shrinkage and Density

In a laboratory experiment we measure the linear change in dimensions of a specimen. The problem here is how to convert this measurement into the change in the relative density. The derivation given below gives a start to the problem, but not the final result.

$$\frac{V_f}{V_o} = \left(\frac{L_f}{L_o}\right)^3 = \left(\frac{L_0 - \Delta L_f}{L_o}\right)^3 = \left(1 - \frac{\Delta L_f}{L_o}\right)^3$$

Derive a relationship between density and shrinkage strain.

If L is the current length, and L_0 is the initial length, then the linear strain is defined as

$$\varepsilon = \ell n_e\left(\frac{L}{L_o}\right) \quad (1)$$

$$\varepsilon_a = \ell n\left[\frac{V}{V_o}\right] \quad (2)$$

$$\varepsilon_a = \ell n\left[\frac{V}{V_o}\right] = \ell n\left[\frac{L}{L_0}\right]^3 \quad (3)$$

$$\varepsilon_a = 3\varepsilon \quad (4)$$

We measure the linear strain ε, and from that we wish to get a measure of the density.

$$\rho_g = \frac{M}{V_0}$$
$$\rho = \frac{M}{V} \quad (5)$$
$$\frac{\rho_g}{\rho} = \frac{V}{V_p}$$

$$\varepsilon_a = \ell n\left[\frac{V}{V_o}\right] = \ell n\frac{\rho_g}{\rho} = 3\varepsilon$$

$\rho = \rho_g e^{-3\varepsilon}$ (6)

Please represent the initial (relative density) by ρ_g where the subscript "g" represents green density. The running density related to the shrinkage strain at any time is written as ρ.

Make a plot of α versus ρ. Write 50 words to explain the salient features of your plot. Comment on the significance of normalizing the pore radius with respect to the grain size (or the particle size).

Assume the particles to be in the shape of a cube with one pore placed at each corner. The particle size if d and the pores, assumed to be spherical in shape have a radius of r.

$$V_{pore} = \frac{4}{3}\pi r^3$$

$$N_d = number\ of\ pores\ per\ grain = 1$$

$$V_{total} = d^3 + N_D\left(\frac{4}{3}\pi r^3\right) \quad (7)$$

$$\mathrm{Re}\,lativeDensity$$

$$\rho = \frac{d^3}{d^3 + \left(\frac{4}{3}\pi r^3\right)}$$

Taking some limits: (i) if the pore is completely filled than r=0, and the density =1. Let us say that the green density is 0.5, then what is the pore radius?

$$\alpha = \frac{r}{d}$$

$$\rho = \frac{1}{1 + \frac{4\pi}{3}\alpha^3} \quad (8)$$

Relating the pore size to the density

alpha	denom	ro
0.10	1.0042	0.9958
0.20	1.0335	0.9676
0.30	1.1131	0.8984
0.40	1.2680	0.7886
0.50	1.5235	0.6564
0.60	1.9046	0.5250
0.70	2.4365	0.4104