# Lecture Notes for Wednesday 04/01/2020

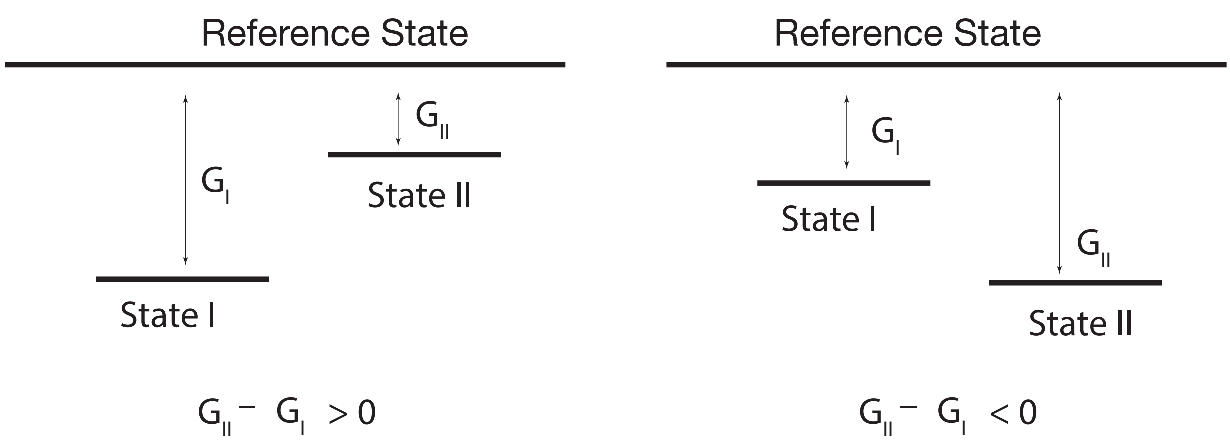
## Topics

(i) Taking Gibbs Free Energy approach to fracture.

(ii) Consider the fracture criterion at the crack tip which can explain why the work of fracture is material dependent (for example glass, which is brittle, versus tool-steel which is semi-brittle) — to be considered on Monday 04/06/2020.

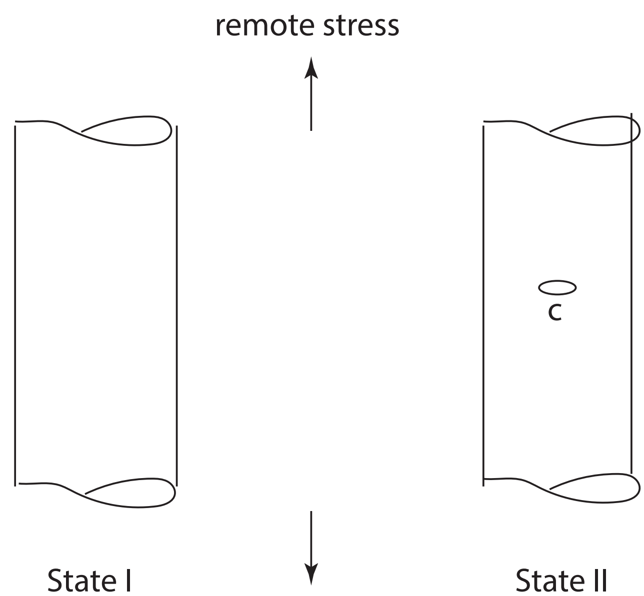
## (i) Taking Gibbs Free Energy ($\Delta G$) approach to fracture.

$\Delta g$ is the amount of work the system can do on the surroundings in going from one state to another state. Two cases where either one of the states is lower than the other are shown in the figure just below




The underlying tenet of Gibbs Free Energy is that a system will lower its energy, if it can, by doing work on the surroundings, thereby achieving a more stable state. It follows that a minimum in Gibbs Free Energy represents an equilibrium state while a maximum represents an unstable state. In the case of the right hand side of the above figure, State II is the more stable state.

We can apply Gibbs Free Energy criterion to crack propagation by considering State I to be without a crack and State II with a penny shaped crack of size "c", as shown in the figure just below

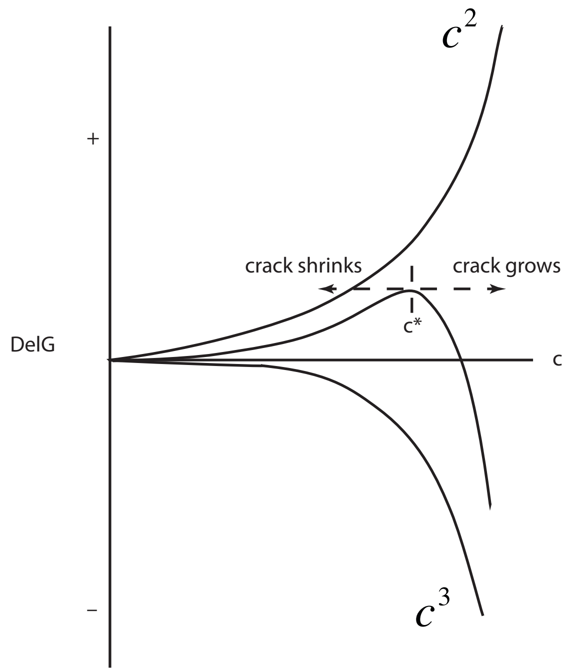



$$\Delta G = G_{II} - G_{I} = +2\gamma_F \pi \left(\frac{c}{2}\right)^2 - \frac{4}{3}\pi \left(\frac{c}{2}\right)^3 * \frac{\sigma^2}{2E} \qquad (2.1)$$

The first term is positive since the crack embodies a surface energy. The second term is equal to the product of the effective volume of the crack multiplied by the energy per unit volume from the applied stress as shown in State I.

The second term represents the change in the mechanical energy. It is a sum of the change in the potential energy and the change in the elastic stored energy. The crack increases the compliance and therefore decreases the potential energy; but it increases the stored strain energy because of the higher stresses in the neighborhood of the crack. Since the magnitude of the potential energy is twice the magnitude of the strain energy, the total mechanical energy will have a negative sign; it can be written as either one half of the potential energy or equal to the change in the stored energy. The second term on the right in Eq. (2.1) represents the change in the stored elastic energy.

The form of Eq. (2.1) is such that the first term which varies as $c^2$ dominates for small cracks and the negative term dominates when the crack is larger. The transition is given by the maximum in $\Delta g$, which is shown in the figure just below.




Therefore, the critical condition for fracture is obtained by differentiating Eq. (2.1) and equating it to zero. Which will give the following result:

$$\frac{d\Delta G}{dc} = +2\gamma_F \frac{\pi}{4} 2c - \frac{4}{3}\frac{\pi}{8} 3c^2 * \frac{\sigma^2}{2E} = 0$$

so that,

$$2\gamma_F = c \; * \frac{\sigma^2}{2E} \qquad (2.2)$$

Therefore (for the case of plane strain):

$$2\gamma_F = \frac{K_{IC}^2}{2E},$$

$$K_{IC} = \sigma\sqrt{c} \qquad (2.3)$$

It can be shown for a thin sheet when the stress normal to the sheet is zero, (the case of plane stress)

$$2\gamma_F = \frac{K_{IC}^2}{E},$$

$$K_{IC} = \sigma\sqrt{c} \qquad (2.4)$$

Practice Homework Problem: Analyze case of an edge crack of length "c" in a thick plate (a plane strain problem) by considering a body with a ribbon shaped crack of width "2c". Here the elastic strain energy will be specified by the cylinder encompassing the ribbon as the effective volume.

$$2\gamma_F = \frac{\pi}{2}\frac{K_{IC}^2}{E},$$

$$K_{IC} = \sigma\sqrt{c} \qquad (2.5)$$

Note that the expression differ by a numerical factor.

Explain that the fracture toughness of the same material would be weaker in plane strain than in plane stress by a factor of $\sqrt{2}$.

In further discussion we shall assume the following form of the relationship between fracture toughness ($K_{IC}$) and the work of fracture $2\gamma_F$, as give by Eq. (2.4)

$$2\gamma_F = \frac{K_{IC}^2}{E},$$

$$K_{IC} = \sigma\sqrt{c} \qquad (2.6)$$

**(i) Fracture Behavior of Different Classes of Materials**

Equation (2.6) relates the resistance to fracture, or the fracture toughness, $K_{IC}$, to the work of fracture, $2\gamma_F$. These parameters have the following significance,

(i) The fracture toughness is an engineering parameter to be used in design and life prediction, for example extrapolating the residual life in an aeroplane wing when there is a visible crack where it meets the fuselage.

(ii) The work of fracture is related to the resistance to crack propagation, for example to the dissipation of work to produce local fracture at the crack-tip. For example the molecular chains in a polymer are more difficult to break at the crack tip that are bonds at a crack tip of a brittle material like glass. In the polymer the chains need to stretch before they break, thus unloading the stress at the crack tip, thereby considerably increasing the work required to propagate the crack.

(iii) The role of the elastic modulus, $E$, it also critical. We have learnt in recent lecture(s) that greater compliance increases the drop in potential energy, and therefore, provide greater mechanical force to propagate the crack. A similar concept applied to the elastic modulus – a lower modulus increases the compliance and lowers the resistance to fracture. So, we note from Eq. (2.6) that even if $\gamma_F$ is large, a low value of $E$, lowers the fracture toughness of the material.

Consider the above pointers in understanding the fracture behavior of different classes of materials, glasses, wood, polymers, metals, composited and ceramics. The "map" on the following page shows plots of $K_{IC}$ against $E$ for several such groups of materials. The following trends are noteworthy,

(i)As a general trend, materials with a higher Youngs Modulus also have a higher fracture toughness, which is in broad agreement with, Eq. (2.6)

(ii) The work of fracture can be shown by drawing lines (on this logarithmic graph) with a slope of one order of magnitude increase in $K_{IC}$ implies two orders of magnitude increase in $E$, with the intercept being related to $2\gamma_F$.

$$2\gamma_F = \frac{K_{IC}^2}{E}, \qquad (2.6)$$

$$K_{IC} = \sigma\sqrt{c}$$

(iii) Note for example that monolithic polymers have the work of fracture at carbon fiber reinforced polymers (GFRP) because the fracture mechanism is similar. But the fracture toughess of GFRP is much higher because they have a higher modulus (as a result of reinforcement with carbon fibers).

(iv) On the other hand metallic composites have a much higher fracture toughness as compared to glass or monolithic ceramics because the work of fracture is much greater for metals.