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Fall 2019, Ceramics (5000/4000 level course)

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Available for consultation on Th and Fr by appointment through email or phone.

•The course is divided into the following topics. The topics concentrate mostly (but not exclusively) on zirconia as the ceramic material of interest.

I. Elastic deformation and fracture
II. Sintering and Superplastic Forming
III. Electrochemical applications: fuel cells, batteries, gas sensors, gas separation and catalysis

•For each topic you will have practice homework questions which you may work on by yourself or with friends. These will not be graded, but I can discuss the answers in class (no written solutions).

•At the end of each topic you will have an in-depth take home HW (essentially like an exam). You will have ten days to submit your answers (e.g. Monday to Friday of the following week).

•Each such take home HW will carry 25% of the grade.

•The remaining 25% will be based upon a 15-20 min presentation on a topic of your choice (towards the end of the semester). I will provide feedback on your presentations starting in early November so please submit your preliminary ppt file by then.

Practice HW 1 Elasticity and Transformation Toughening

Attention: Please note that this HW will not be graded. I can discuss in class but written solutions will not be provided

1. Show that the volumetric strain $\varepsilon_a = \frac{\Delta V}{V}$ is given by:

$$\varepsilon_a = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

Where the quantities on the right are the principal strains.

2. Consider a simple uniaxial tensile test where the response is related to the Youngs Modulus and the Poisson's Ratio. Show that the B, the bulk modulus is related to E and ν by

$$B = \frac{E}{3(1-2\nu)}$$

The Bulk modulus is defined as:

$$p = -\frac{\Delta V}{V} B$$

Where p is the pressure (equal to the sum of the diagonal terms in the strain tensor divided by 3).

3. Explain why the elastic strain is a better, that is, a more universal, measure for prescribing fracture (or the onset of plastic yielding in metals).

4. The work done during elastic deformation is generally the sum of two quantities: the change in the potential energy and a change in the stored elastic energy. Show that the magnitude of the first is always twice the magnitude of the second, and that the two quantities are opposite in sign.

5. Show that the incremental expansion of small crack in an elastically deforming body always leads to the consumption of energy, whether the experiment is carried out under constant load or constant displacement.

6. The force displacement relation for a cantilever of length L, width w and thickness h, with a load P applied to the free end is given by

$$y = P^{*} \frac{4L^3}{3Ewh^3} \qquad (1)$$

Note that P^{*} here has units of Newtons. The force per unit width of the beam is then

$$P = \frac{P^{*}}{w} \qquad (2)$$

Now consider a double cantilever beam specimen used to measure the work of fracture of the material. In this instance the crack length, c, is equal to L. Further the force per unit length of the crack front is given by the Eq. (2) just above.

If fracture occurs at $P = P_{crit}$ then show that the work of fracture is given by

$$2\gamma_F = \frac{P^2}{2}\frac{dS}{dc}$$

where S is the compliance.

Remember that in a double cantilever beam the total displacement is twice the displacement given by Eq. (1).

7. Consider a penny shaped crack, of diameter $2c$, in an elastically deforming body which is loaded with a uniaxial tensile stress, σ. Derive that

$$2\gamma_F = \frac{2K_{IC}^2}{2E} \qquad (3)$$

Where $K_{IC} = \sigma_{crit}\sqrt{2c}$. Recall that the excess elastic stored energy for a penny shaped crack is given by

(energy per unit volume remote from the crack which is $\frac{\sigma^2}{2E}$)x(effective volume of the crack $\frac{4}{3}\pi c^3$)

8. Equation (3) above is often used to describe the work of fracture even when there is local deformation near the crack tip, for example in crack propagation in tool steels, or transformation toughening in zirconia. State a physical limit to the application of linear elastic fracture mechanics to these situations.

9. Give the reason why the fracture toughness measured in plane strain experiments is always lower than the fracture toughness measured in plane stress experiments.