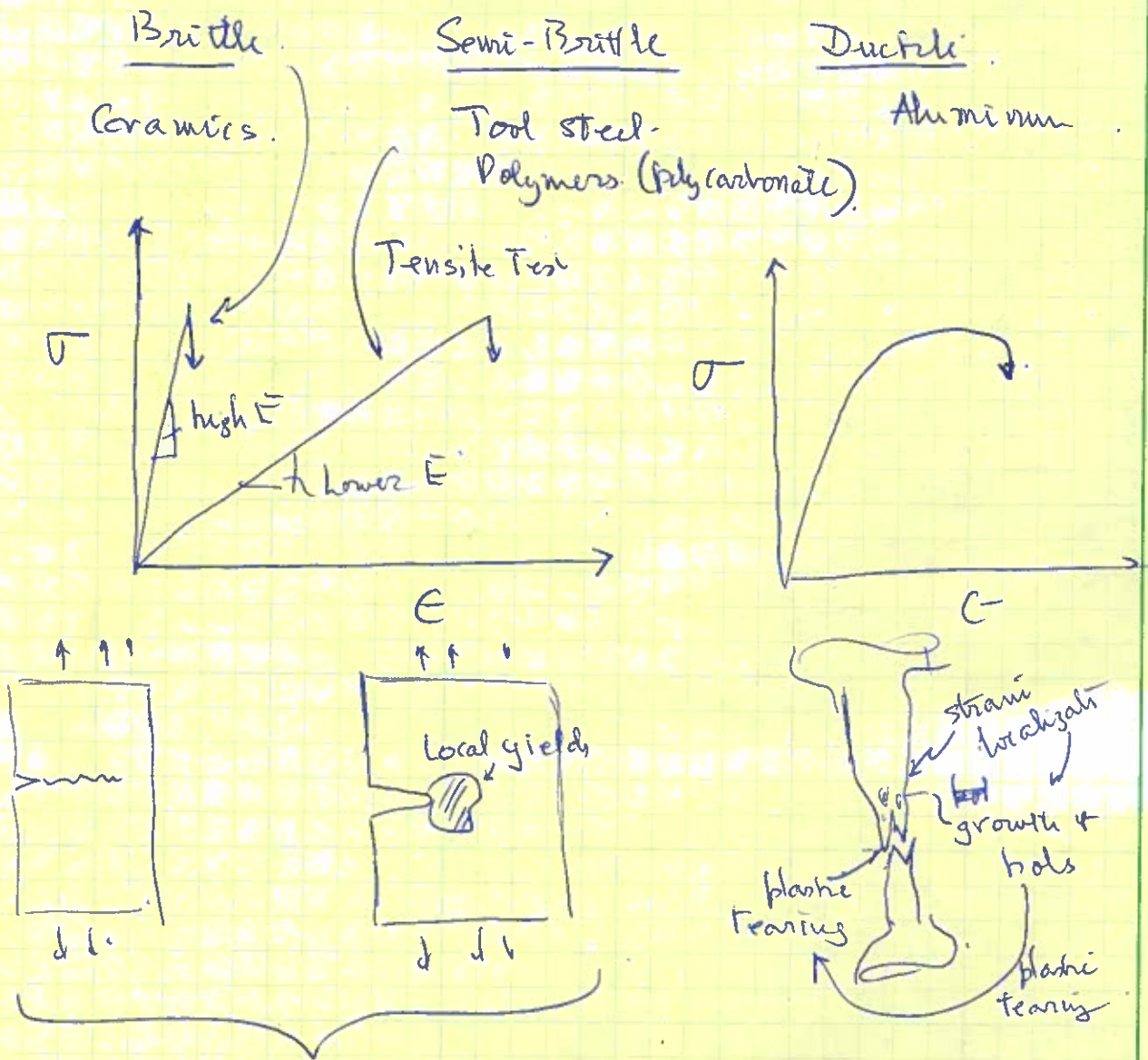


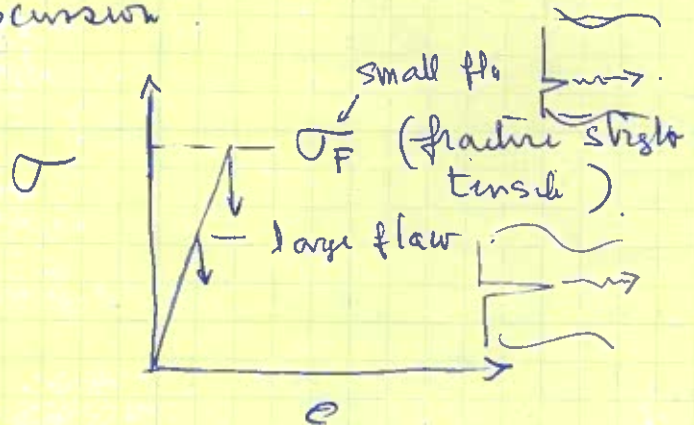
What is fracture?



Topics of Discussion

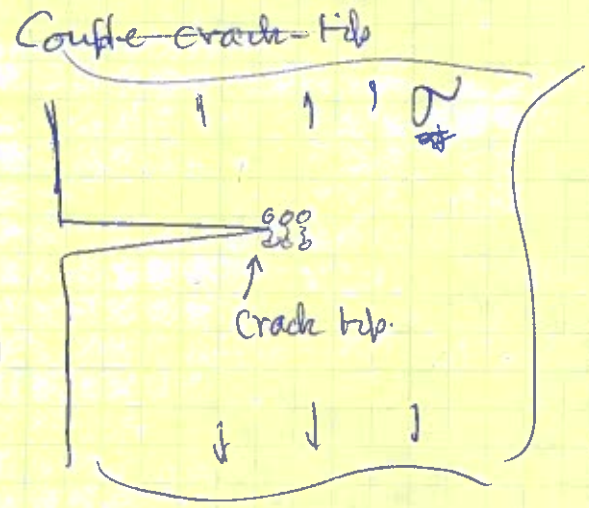
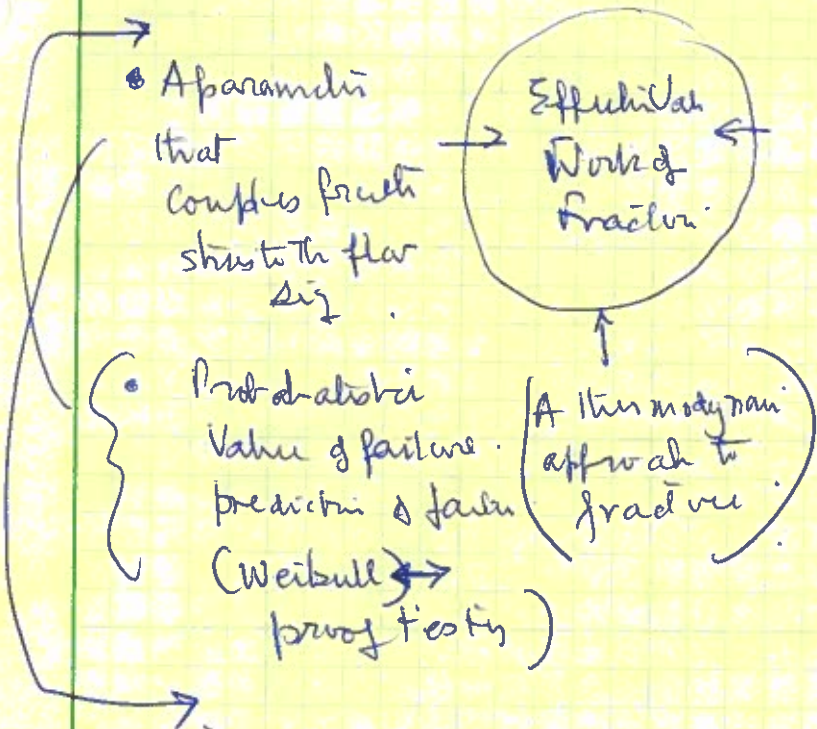
BRITTLE FRACTURE

σ_F is NOT a material property \rightarrow depends on its "flaw size"
 ↳ microcrack



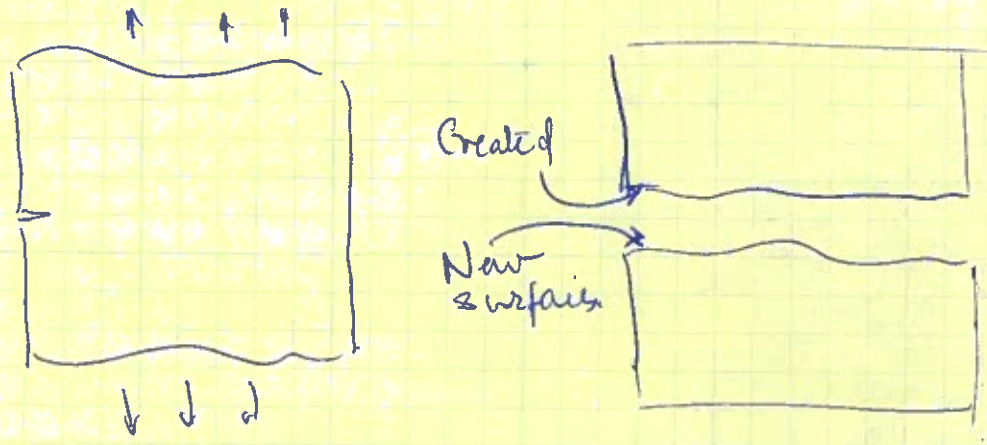
Questions
Mechanics

Material Sci



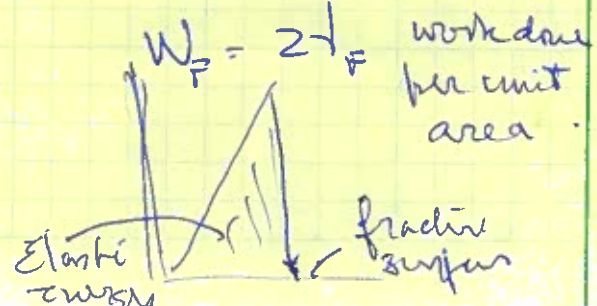
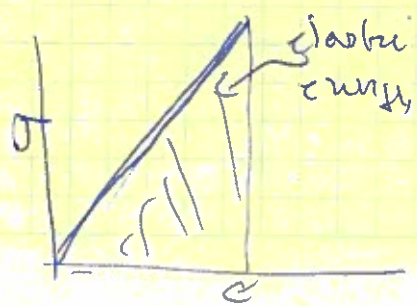
Couple the local fracture criterion at the crack tip with the applied stress.

The Work of Fracture

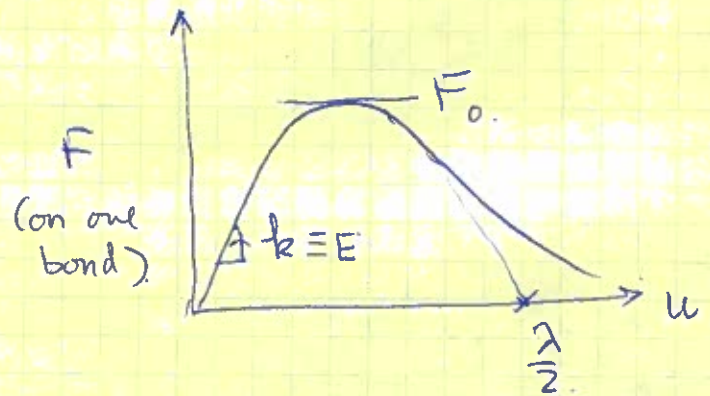
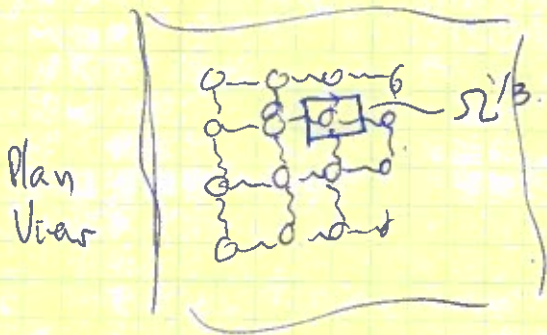
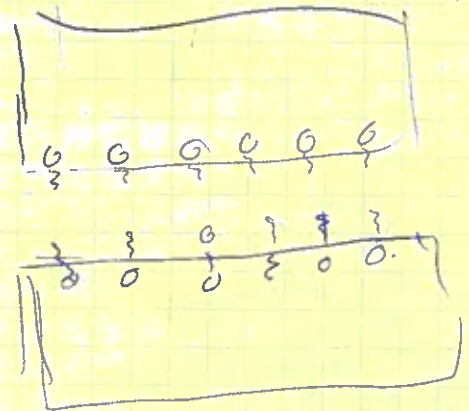
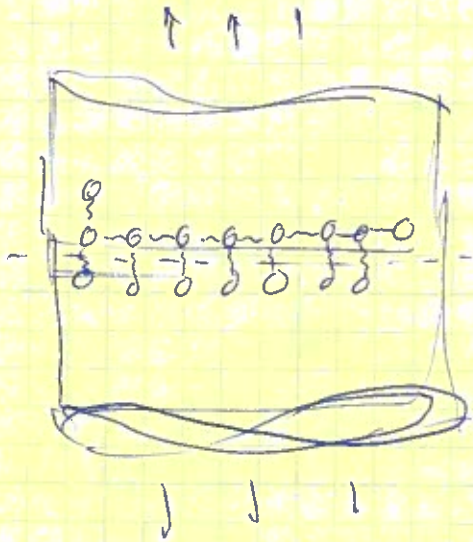


State I

State II



Conceptualize a simple bond breaking model in an ideal crystal
 i.e. without any defects:



$$F = F_0 \sin \frac{2\pi u}{\lambda}$$

$$k = \left(\frac{dF}{du} \right)_{u \rightarrow 0} = \frac{2\pi F_0}{\lambda}$$

$$\frac{\lambda}{4} = 0.25 \Omega^{1/3} \quad \sigma_F = \frac{F_0}{\Omega^{2/3}} \quad k = \frac{k}{\Omega^{1/3}} = \frac{2\pi F_0}{\lambda \Omega^{1/3}}$$

$$\sigma_F = \frac{E \lambda \Omega^{1/3}}{2\pi \Omega^{2/3}} = \frac{E}{2\pi} \sim \frac{E}{10} \quad (*)$$

$$\epsilon_s = 10\%$$

Work of fracture (per bond), $D_{fr} \gamma_F$.

$$2 \gamma_F \Omega^{2/3} = \int_0^{\lambda/2} F du$$

$$= \int_0^{\lambda/2} F_0 \sin \frac{2\pi u}{\lambda} du$$

$$= - \frac{\lambda}{2\pi} \times F_0 [-1 - \cos(1)]$$

$$= \frac{\lambda}{\pi} F_0$$

$$\lambda = \Omega^{1/3}$$

$$F_0 = \frac{E \lambda \Omega^{1/3}}{2\pi}$$

$$2 \gamma_F \Omega^{2/3} = \frac{\lambda}{\pi} \times E \frac{\lambda \Omega^{1/3}}{2\pi}$$

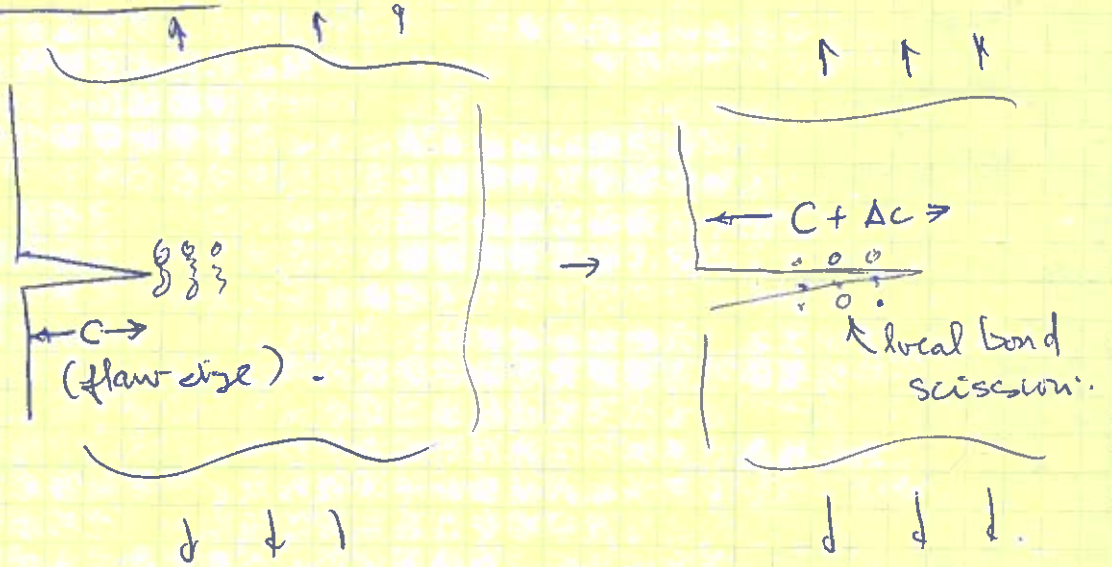
$\Omega^{1/3} = a$
 lattice parameter

$$\gamma_F = \frac{Ea}{4\pi^2}$$

~~$$\gamma_F = \frac{Ea}{4\pi^2}$$~~

Ideal crystal \rightarrow no flaws \rightarrow ideal fracture strength

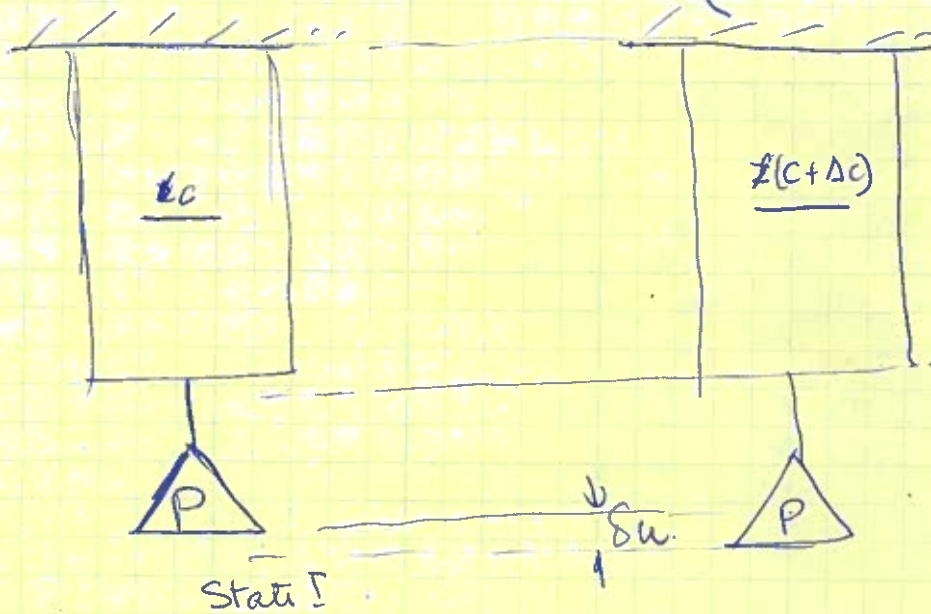
More realistically



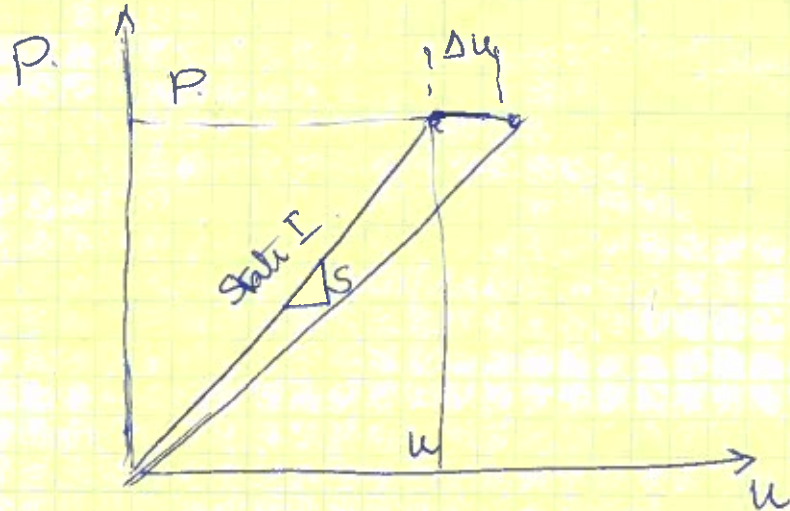
Thermodynamic Approach to Fracture.

Global \rightarrow work is done by the applied stress.

Local \rightarrow Work of fracture related to bonds breaking.



2D problem \rightarrow a ribbon shaped flaw



Change in Mechanical Energy

~~Work done by the system in~~

~~Reduction in the~~

Change in the Potential Energy.

Change in the Stored Elastic Energy

State I → State II

$$- P \Delta u$$

$$\frac{1}{2} P(u + \Delta u) - \frac{1}{2} P u$$

$$= \frac{1}{2} P \Delta u$$

Note the total change →

$$= - \frac{1}{2} P \Delta u = - \left| \begin{array}{l} \text{change in the} \\ \text{stored elastic} \\ \text{energy} \end{array} \right| = - \left| \begin{array}{l} \text{change in the} \\ \text{potential} \\ \text{energy} \end{array} \right|$$

$$2\gamma_f \Delta c \leq \frac{1}{2} P \Delta u.$$

$$S = \frac{u}{P}$$

$$\Delta S = \frac{\Delta u}{P}$$

$$\leq \frac{1}{2} P^2 \cdot \Delta S$$

$$\boxed{2\gamma_f \leq \frac{P^2}{2} \frac{ds}{dc}}$$

Gibbs Free Enrgy Approach

$$\Delta G = -2\gamma_f c + \frac{1}{2} P u.$$

$$\frac{d\Delta G}{dc} = -2\gamma_f + \frac{1}{2} P \frac{du}{dc} \leq 0.$$

$$S = \frac{u}{P}$$

$$\frac{du}{dc} = P \frac{ds}{dc}$$

→ same result obtained by differentiation.