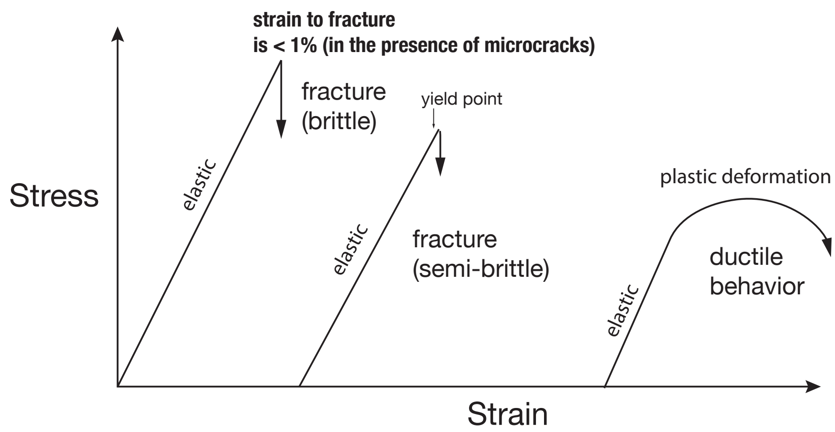
04B: Global Theory of Fracture

Parametric Description of the Fracture Phenomenon



•Elastic Deformation is described by the Youngs Modulus $E = \dfrac{\sigma}{\varepsilon}$

•Plastic Deformation is described by the "Yield Stress" (usually in metals only)

•Fracture is described by fracture toughness, K_{IC} with units of MPa $m^{1/2}$

$K_{IC} = (\text{geometrical factor})\,\sigma\sqrt{c}$ MPa $m^{1/2}$. Typical value for polymers and ceramics is in the range of 1 – 5 MPa $m^{1/2}$. K_{IC} is called the fracture toughness. (1)

However, the work of fracture is a fundamental quantity: it is the work done to extend the crack by one unit area. Since cracks have surfaces, the work may therefore be written in as "$2\gamma_F$", which has units of J m^{-2}. However, γ_F is not a surface energy, it is the work of fracture per units area of the fracture surface. The factor of two recognizes that the propagation of a crack creates two surfaces.

We will show that

$$2\gamma_F = \frac{K_{IC}^2}{E} \qquad (2)$$

Thus, equations (1) and (2) are fundamental to the study of fracture. While K_{IC} serves as the engineering parameter in mechanical design. The work of fracture, $2\gamma_F$ is related to the materials science, and the atomistic mechanisms of fracture, as were described in the overview.

The Global Theory of Fracture

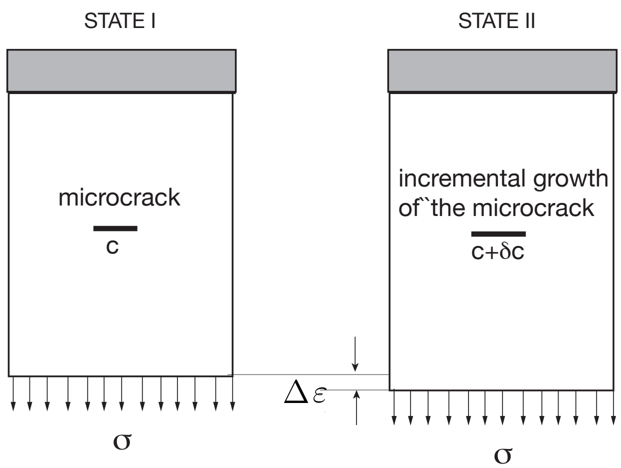
The global theory of fracture rests on the following principle,

"**The objective of the theory** is to obtain the critical stress for fracture for a given flaw size as epitomized in Eq. (1) for fracture toughness.

The propagation of cracks required work ($2\gamma_F$); this work is provided by a combination of stored elastic energy and the change in the potential energy of the system. The critical condition is obtained when the work done on the system can be greater than the work of fracture."

The Approach

Energy consideration for incremental growth of a microcrack



What is the complexity?

(i) The applied stress does work on the system. Imagine a load applied to the specimen. The higher compliance in the presence of the crack will cause the load to drop a little thus losing potential energy. This loss in potential energy is the work that the load does on the system.

(ii) On the otherhand, the stress concentration around the crack will lead to a higher strain energy in the material surrounding the crack.

We will show that (ii) is exactly one half of (i) in magnitude. However (i) is a negative quantity (since the energy of the system is lowered) while (ii) is a positive quantity since it raises the energy stored in the system.

Therefore, the total change in energy can be written either as

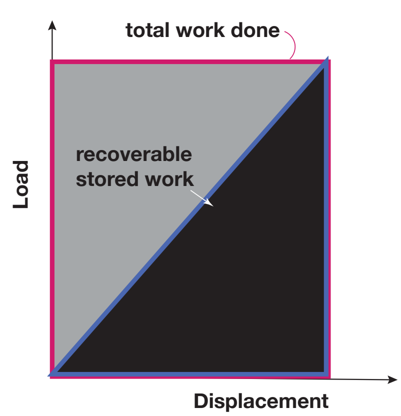
(a) $-\left|\text{magnitude of (ii), i.e.the excess strain energy}\right|$

or

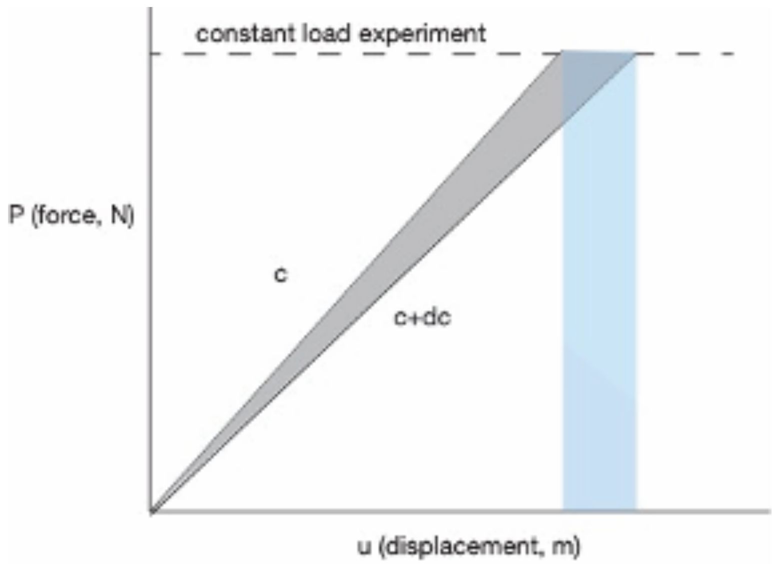
(b) $\frac{1}{2}$reduction in the potential energy in (i) (it is already a negative quantity.

The simplification given just above can be explained by the linear nature of the load - displacement diagram, as shown on the right.

Note that the total work done is the area in the red rectangle. However, only one half of that work is stored as elastic energy.



Now consider the change in the load displacement curve for the diagram as shown above where the crack grows incrementally,



The growth of the crack from c to c+δc produces a small increment in external displacement. The work done by this displacement on the system is the area in the blue rectangle. If the experiment is done at a constant load then this work is equal to $-P\delta u$, with the minus sign representing that the decrease in the potential energy of the system decreases, this energy is available to grow the crack.

The increase in compliance also leads to an increase in the stored elastic energy, which is equal to the difference in the area of the two triangles colored in grey. This contribution increases the stored energy, and, therefore, opposes crack growth - it is therefore a positive quantity.

Simple geometric construction shows that the area of the grey triangle is one half the area of the blue rectangle.

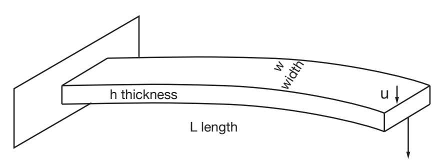
Therefore, the total "magnitude" of the change in energy for the incremental growth of the crack can be written as one half of the area of the blue rectangle or equal to the area of the grey triangle. This quantity, the total change in energy is a negative value.

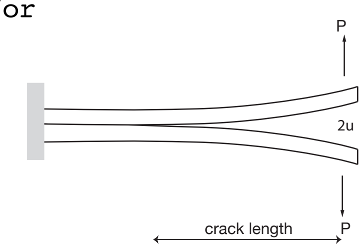
The upshot of the above theorem is that we just need to calculate the increase in the stored energy when the crack grows incrementally even if the crack is contained within a large body with a complex set of applied forces to the surface of the body.

The DCB (Double Cantilever Beam Experiment)

The equations for the force/displacement relationship for a single cantilever beam are given on the right.

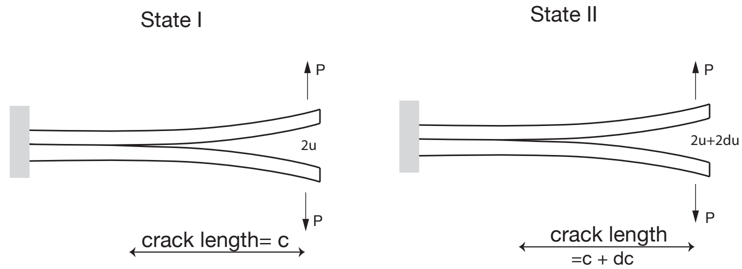
The DCB specimen will have twice the displacement shown as 2u (for the same load P applied to both wings of the two beams.

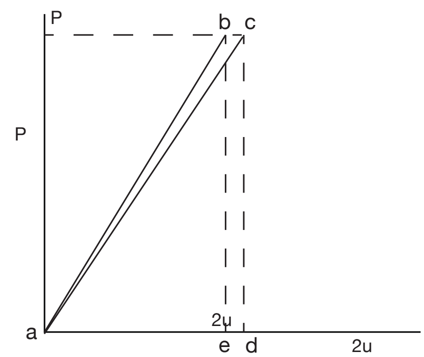




$$u = \frac{PL^3}{3EI}, \; I = \frac{wh^3}{12} \qquad k = \frac{P}{2u}, \; S = \frac{2u}{P}$$

The DCB (Double Cantilever Beam Experiment)





We consider two states. In the first the crack length is equal to "c" while in the second the crack is assumed to grow incrementally to "c+δc".

If the total change in energy from State I to Stage II is negative, that is the energy is lowered when the crack advances, then the crack will grow.

The change in energy consist of three terms:

(i) the increase in the surface energy of the crack (a positive contribution)

(ii) the work done on the system by the applied load (a negative contribution)

(iii) the increase in the stored elastic energy (a positive contribution)

As discussed on the previous page, magnitude wise (ii) is equal to two times (iii). Therefore, the total of (ii)+(iii) is negative. Its magnitude can be set equal to one half of (ii) or to (iii).

The geometrical parameters for the DCB specimen are as follows

•The length of the free section of the beam is also the crack length; therefore "L" in the previous figure is now "c".

•The width of the beam, normal to the plane of the paper, is "w"; note that this is also the perimeter of the crack- sometimes written as the edge-length of the crack.

•The thickness of the beam is given by "t".

The first term (inside the box) is given by

$$2\gamma_F w \delta c \qquad (3)$$

This is the increase in the surface energy of the crack as it grows from "c" in State I to "c+δc" in State II.

The sum of the second and the third terms inside the box may be written as one half of the area of the rectangle "bcde" in the figure above. It is equal to

$$\frac{1}{2}P(2\delta u) \qquad (4)$$

This term has a negative sign.

The fracture criterion, that is the critical stress for the propagation of the crack can now be written as

$$2\gamma_F w\delta c - \frac{1}{2}P(2\delta u) \le 0 \qquad (5)$$

The next step is to relate δu to the change in the compliance of the specimen as crack grows. The compliance, S has units of m/N, and is written as

$$S = \frac{2u}{P}$$
$$\frac{dS}{dc} = \frac{1}{P}\frac{d(2u)}{dc} \qquad (6)$$

Combining Eqns (5) and (6) we obtain, that crack will propagate at a load P^* when

$$2\gamma_F = \frac{P^{*2}}{2w}\frac{dS}{dc} \qquad (6)$$

Let us check the units

P has units of load, i.e. N.

S has units of displacement divided by the load, i. e. m N^{-1}

Therefore, RHS has units of N m^{-1},

i.e. (N*m) m^{-2}, or Jm^{-2}

Notes:

(a) The quantity $\frac{dS}{dc}$ can be measured by preparing several specimens with different crack length cut into them. The compliance for each of them can be measured from the slope of the data for displacement versus load as shown on the right. A plot of S versus C will then yield a value for $\frac{dS}{dc}$.

(b) In the experiment the critical load to fracture, P^*, is measured for a given value of "c", which when inserted into Eq. (6) will give the value for $2\gamma_S$

(c) Note how the fracture property of the materials is measure without and mechanics analysis. This simplicity off the experiment was made by the fact that the load displacement plots were assumed to be linear.. so that the energy stored in the specimen was elastic energy.

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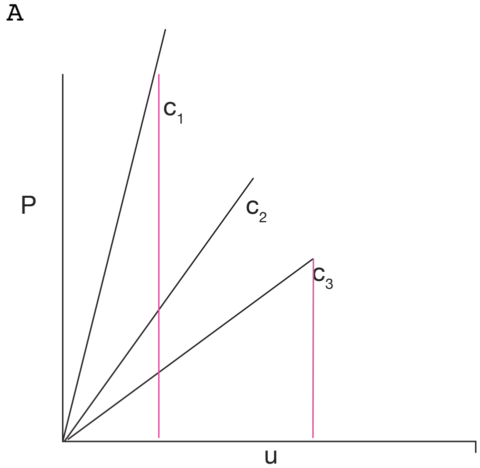
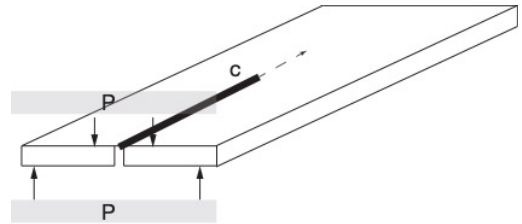


Alternatively the equations for beam flexion, which are given above can be used to simulate an experiments for silica glass. Using $\gamma_S = 1\ \mathrm{J\,m^{-2}}$ and reasonable values for the DCB specimen calculate critical load to facture (it should be a few kg). The elastic modulus for glass is 80 GPa.

Generality of the above method to other specimen configurations

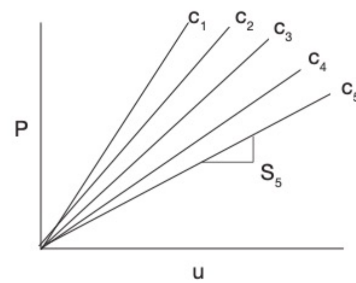


Method

•Make five specimens with different lengths of "c" cut into them.

•Measure the compliance for each of them

•Make a plot of compliance against "c" to obtain (dS/dc)

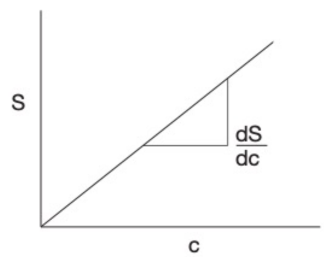


Now load the crack to fracture to determine P* for a given crack length. Substitute in

$$2\gamma_F \geq \frac{P^{*2}}{2h}\frac{dS}{dc},$$

to obtain the fracture energy.

Here "h" is the thickness of the double torsion experiment .



Summary

In this chapter we have shown how the work of fracture, expressed as surface energy in units of J m^{-2} can be determined from simple load displacement curves of a specimen. Two measurements must be made (i) the load at which a crack of a given length propagates, and how the compliance of the specimen changes with the crack length. These measurements are possible simply from experiments without the need for analysis of the mechanics of deformation. However, for the case of a DCB specimen the well known result for elastic deformation of beams can be used for the determination of $\frac{dS}{dc}$.