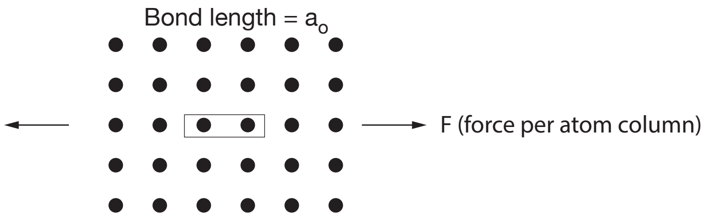
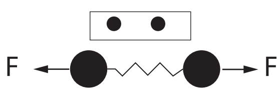
# 04C: The Stress Intensity Factor ($K_I$)

# Background

## Elastic deformation

In elastic deformation all the bonds are stretched equally, that is the strain is uniform throughout the body. Therefore, the strain measured in the laboratory with a physical specimen is also the strain at the atomic level




$$\text{tensile strain} = \frac{u}{a_o}$$

$$\text{tensile stress} = \frac{F}{a_o^2}$$

Therefore, the Youngs Modulus

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{a_o^2}}{\frac{u}{a_o}} = \frac{F}{u}.\frac{1}{a_o} = (\text{bond stiffness}).\frac{1}{a_o}$$

Elastic properties are defined by the stiff of the individual bonds.

As we have done before we write the interatomic distance as the cube root of the atomic volume, that is

$$a_o = \Omega^{1/3}$$

Approximately we can say that $\rho(\text{the density}) \propto \frac{1}{\Omega}$ (not exactly correct since the mass can change significantly)

Then we can say that

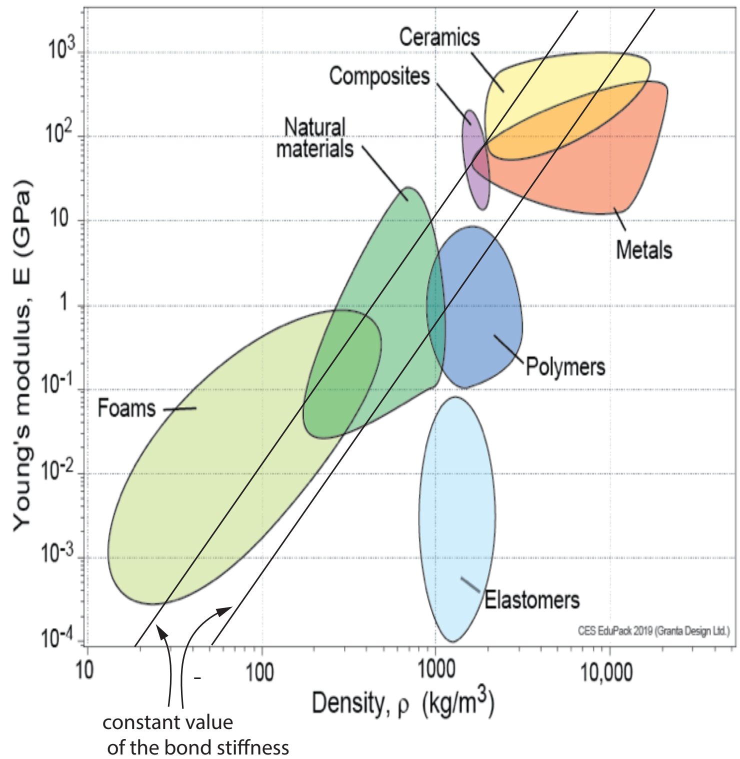
$$\frac{E}{\rho^{1/3}} = \text{bond stiffness}$$

BOND STIFFNESS INCREASES IN THE FOLLOWING ORDER

- Polymers
- Natural materials like wood
- Glass - soda lime
- Glass - silica
- Metals
- Refractory metals (high melting temperature)
- Ceramics

$$\frac{E}{\rho^{1/3}} = \text{bond stiffness}$$

Lines for a constant value of the bond stiffness.

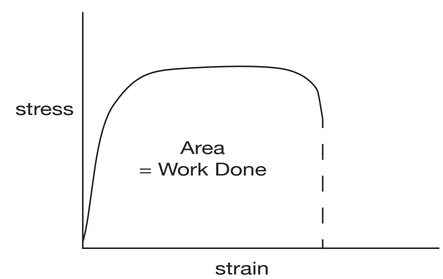



## Plastic Deformation (metals)

Deformation is localized.

It is characterized by a yield stress, that is the applied stress for the onset of plastic deformation.

At the atomic level it is produced by collective movement of atoms which produces a change in shape without changing the structure of the crystal.

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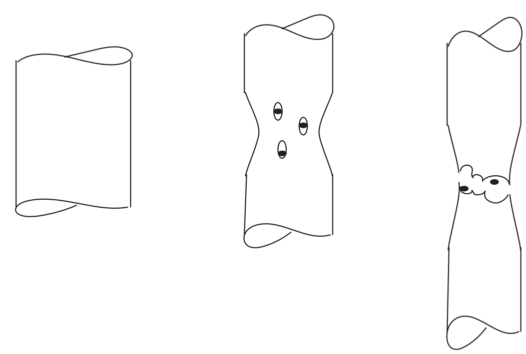


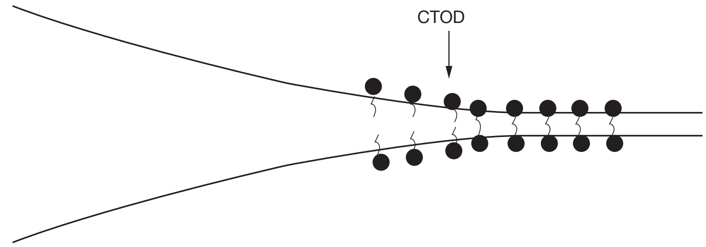
## Fracture

Bonds rupture, or the material "tears apart" at the crack tip.

The loading at the crack tip is now defined by a quantity called,

The Stress Intensity Factor $K_I = \alpha\sigma\sqrt{c}\ \text{MPa}\sqrt{\text{m}}$

Here $\alpha$ is a factor of order unity which depends on the geometry (penny versus an ellipse for example) and the orientation of the crack relative to the tensile axis.

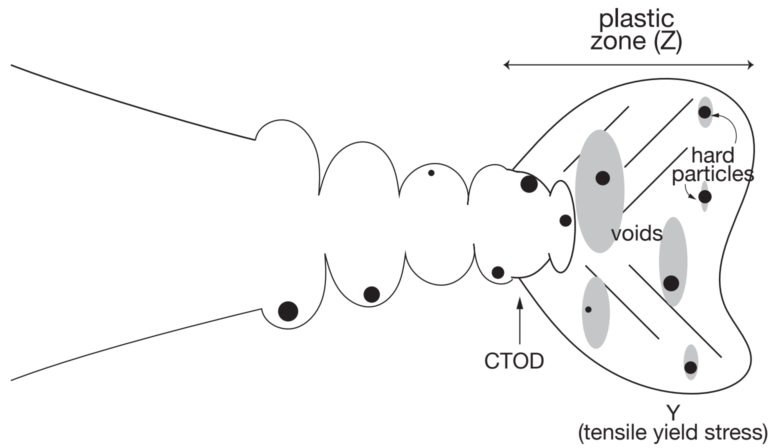



If the deformation at the crack-tip is entirely elastic, as shown in the diagram on the top right), then the displacements and stresses near the crack tip can be written in terms of $K_I$

The fracture condition is defined when

$$K_I \geq K_{IC}$$

Therefore $K_{IC}$ now becomes a materials property, equivalent to the elastic modulus for elastic deformation and the yield stress for plastic deformation.



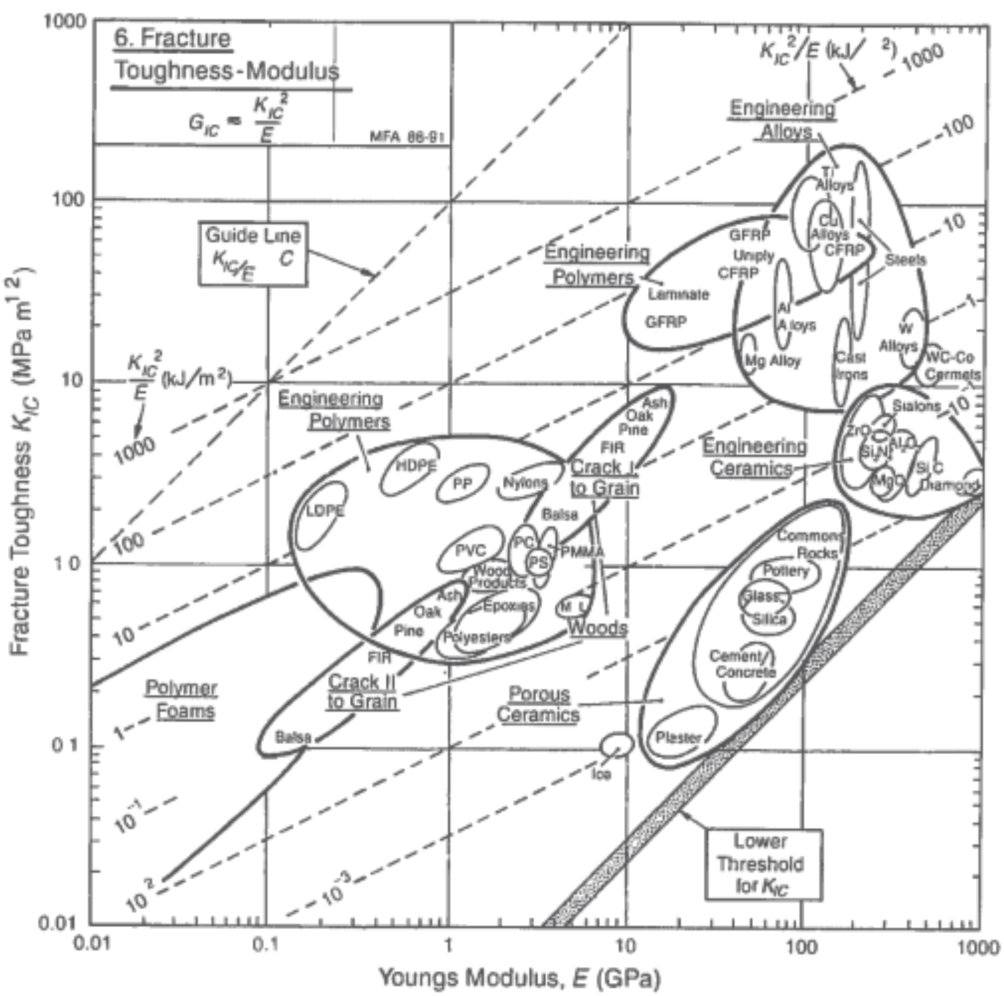

We can further relate the fracture toughness, $K_{IC}$ to the work of fracture by the following equation (we shall derive it in a moment)

$$2\gamma_F = \frac{K_{IC}^2}{E}$$

The map on the right shows lines for constant values of $2\gamma_F$.

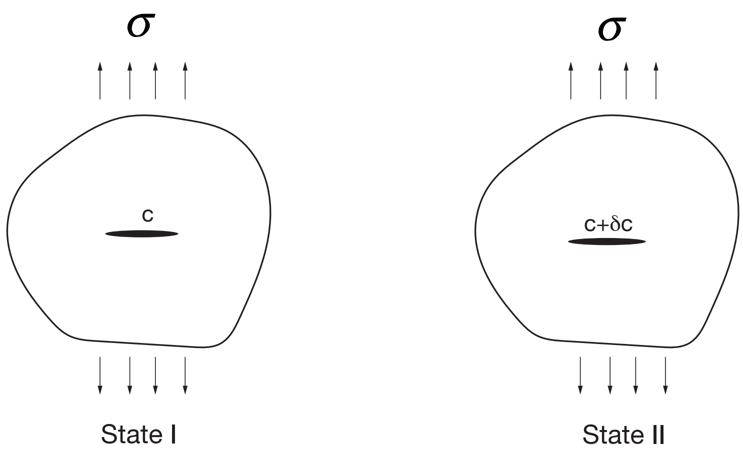
In summary:

- The work of fracture is a fundamental material property. For example its value is about the same for polymers as it is for metals.
- However, $K_{IC}$ which is also a material property (it is a handbook value) it is mutated by the role of the Elastic Constant in the propagation of cracks.



# Derivation of

$$2\gamma_F = \frac{K_{IC}^2}{E}$$




Increase in the fracture surface energy =

$$2\gamma_F \delta\left(\pi\left(\frac{c}{2}\right)^2\right) = 2\gamma_F \frac{\pi}{4}.2c\delta c \quad \text{(positive term)} \qquad (1)$$

Increase in the stored elastic energy is given by

2*effective volume of the crack*remote elastic energy per unit volume.

$$2 * \frac{4\pi}{3}(\frac{c}{2})^3 * \frac{\sigma^2}{2E}$$

Therefore the change in the stored energy from State I to II is given by

$$\delta\left[2 * \frac{4\pi}{3}(\frac{c}{2})^3 * \frac{\sigma^2}{2E}\right] = \frac{\pi}{3} * 3c^2\delta c * \frac{\sigma^2}{2E} = \pi c^2 \frac{\sigma^2}{2E}\delta c \qquad (2)$$

The total of the change in the potential energy and the increase in the stored energy is equal to the -(stored energy)

So now have a fracture criterion

Eq. (1) < Eq. (2) gives the fracture condition

$$2\gamma_F \frac{\pi}{4}.2c\delta c \le \pi c^2 \frac{\sigma^2}{2E}\delta c \qquad (3)$$

$$2\gamma_F = \frac{\sigma^2 c}{E}$$

We say that $K_{IC} = \sigma\sqrt{c}$ , then we have that $2\gamma_F = \frac{K_{IC}^2}{E}$