04D: Fracture at the Crack Tip

Background

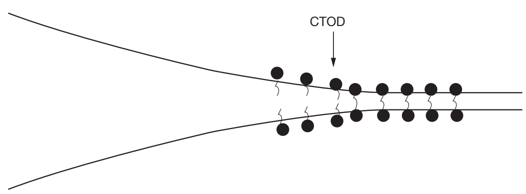
Last time we derived the following equations. The first relates the stress intensity factor to the tensile uniaxial stress and the "flaw size" and the second relates the critical value of the stress intensity factor to the work of fracture per unit area of the crack surface.

> We say that $K_{IC} = \sigma\sqrt{c}$, then we have that $2\gamma_F = \frac{K_{IC}^2}{E}$

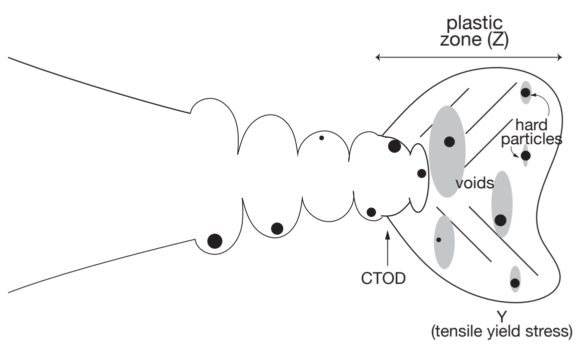
A natural implication of these equations is that K_I is the loading parameter in brittle fracture (just as uniaxial stress is in elastic deformation).

Since fracture must occur by propagation of a crack, and since the work done to propagate the crack is embedded in $2\gamma_F$ (and K_{IC}), these parameters would be related to the fracture criterion at the crack tip.

We shall discuss two types of "fracture mechanisms" at the crack tip - for brittle fracture and for small scale deformation and fracture at the crack tip, as in polymers and high strength metals.



Brittle Fracture



SemiBrittle of fracture with small scale yielding at the crack tip.

Brittle Fracture

The concept is to define a criterion for breaking bonds at the crack tip, and applying that criterion to the stress felt at the crack tip. Instinctively we know that the stress at the crack tip must be related to K_I but only if all deformation is purely elastic. Elastic analysis poses a well-defined problem in elasticity theory, which we expect to give us relationships between local stresses and displacements at the crack tip in terms of K_I.

Here are equations for crack tip stresses and displacements using the coordinates (r,θ) where the center point is located at the crack tip.

Notes:

(i) The stresses and displacements near the crack tip are given in terms of K_I, which is the engineering parameter for loading the crack.

(ii) When $\theta = 0$ then

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$

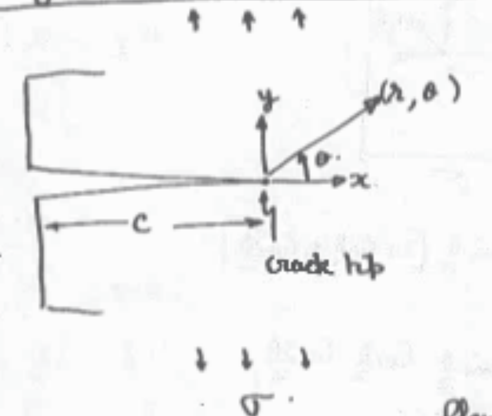
"note a square root singularity". However, the minimum meaningful length scale in the problem is the interatomic distance.

(iii) When $\theta = 0$ then expect that $u_{yy} = 0$ which is what is predicted from the equations on the right hand side.

Question: how do I relate the equations given here to the fracture mechanism for brittle fracture, which is breaking of bonds at the crack tip without any plastic deformation?

Answer: There is a limit to the displacement, which is equal to the stretch in the bonds in front of the crack tip, at which point the bond breaks. The question is how much tensile stress, σ^*_{yy} can a bond sustain before it breaks.

Summary of Mode I Crack Stress Field



Plane strain

$$K_I = \sigma\sqrt{\pi c}$$

$$\sigma_{xx} = \frac{K_1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{yy} = \frac{K_1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{xy} = \frac{K_1}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$ for plane strain

$$u_x = \frac{K_1}{G}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(1 - 2\nu + \sin^2\frac{\theta}{2}\right)$$

$$u_y = \frac{K_1}{G}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(2 - 2\nu - \cos^2\frac{\theta}{2}\right)$$

$u_z = 0$

Plane stress

$\sigma_{zz} = 0$

$\nu \xrightarrow{\text{replace}} \frac{\nu}{1+\nu}$

$E \rightarrow E\frac{(1+2\nu)}{(1+\nu)^2}$

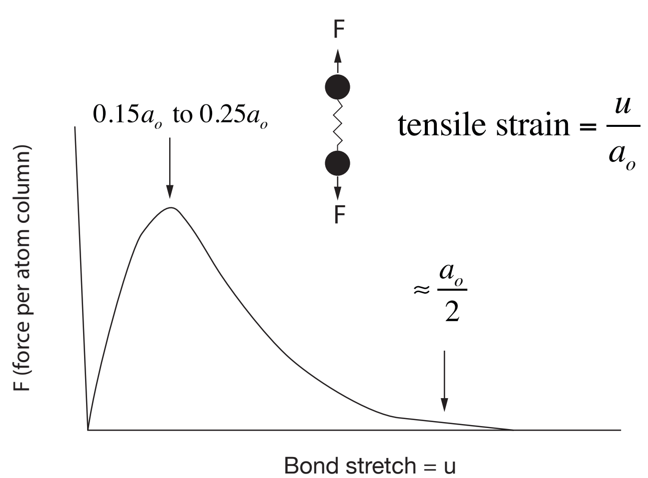
K_I unchanged

$$\sigma^*_{yy}(\theta = 0, r = a_o \; or \; \Omega^{1/3}) = E\,\frac{\text{stretch at the first atom near the crack tip}}{a_o} \quad (1)$$

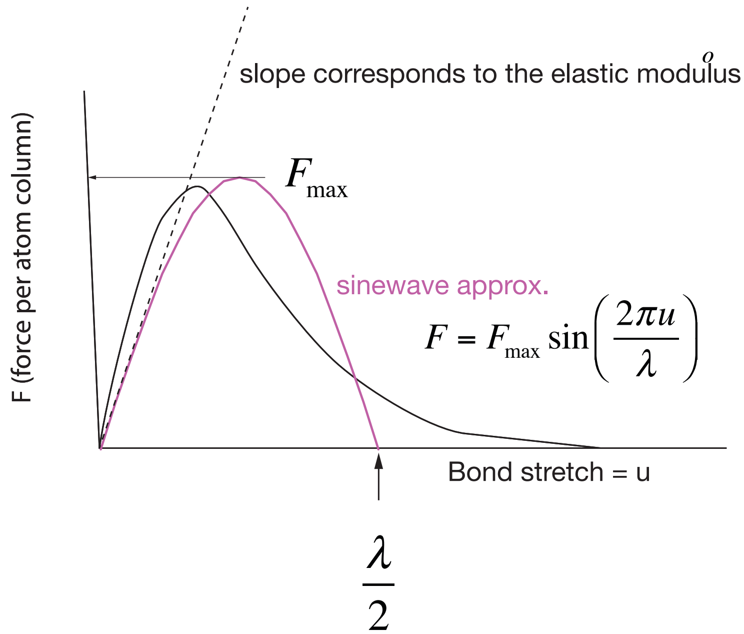
where E is the elastic modulus.

Let us make physical arguments to get an answer to the above equation.

Note the force displacement curve on the right. It rises linearly at first, like spring, but then reaches maximum value when the bond breaks, and then it declines to zero. Note that the force does NOT drop immediately to zero, but rather falls gradually down to zero until the bond is totally separated.



Approximate the force displacement curve by a half sine wave.



Question: what may be the wavelength of this sine-wave?

Answer by asking another question to ourselves, that is, how far must atoms move away from their neighbor until the interaction between them reaches the maximum value, i.e. the peak in the sine wave.

Approximately we say that the force reaches a maximum when

$$u^* = \alpha a_o \qquad (2)$$

this now becomes the fracture criterion at the crack tip. We know that u^* will scale with a_o, and we say the fracture occurs when the displacement is some fraction of the interatomic spacing, this fraction is written as α.

$$\sigma_{yy}^*(\theta = 0, r = a_o \, or \, \Omega^{1/3}) = \alpha E \qquad (2)$$

How do we get and experimental assessment of α?

Well, we measure the fracture stress of IDEAL materials, for example an optical fiber which has no flaws and therefore fracture at the critical value of u^*.

These experiments with near perfect glass fibers give values

$$0.15 < \alpha < 0.25 \qquad (3)$$

By combining the handwritten equations with Eqn (1), (2) and (3), it is possible to obtain an expression for K_{IC} which can be compared with experiment.

Expect just from units and scaling that

$$K_{IC} \propto E\sqrt{a_o} \qquad (4)$$

Let us build on Eq. (2) from the handwritten results,

$$\sigma_{yy}^*(\theta = 0, r = \Omega^{1/3}) = \frac{K_{IC}}{\sqrt{2\pi\Omega^{1/3}}} \qquad (5)$$

Combining (2) and (5) we obtain the final result,

$$\sigma_{yy}^*(\theta = 0, r = \Omega^{1/3}) = \frac{K_{IC}}{\sqrt{2\pi\Omega^{1/3}}} = \alpha E \text{; so that } 75\, K_{IC} = \sqrt{2\pi\Omega^{1/3}}(\alpha E) \qquad (6)$$

Let us apply to silica glass: E = 75 GPa, Mol wt. = 60 g/mol, density = 2.65 g/cm^3, N_A =6.03E+23 #/mol: which gives $\Omega^{1/3}$ = 3.35E-10m.

Substituting into Eq. (6) and taking the limits for α as in Eq. (3), we obtain that:

0.15	5.16E-01	0.25	8.60E-01
0.2	6.88E-01	0.3	1.03E+00

here the first column is the value for alpha and the second column the prediction for K_{Ic} in MPa m$^{1/2}$

The experimental value for fracture toughness of glass ranges from 0.6 to 0.8 MPa m$^{1/2}$ which gives that α is approximately 0.2.