04E: Fracture criterion at the crack-tip

Overview

We say that $K_{IC} = \sigma\sqrt{c}$, then we have that $2\gamma_F = \frac{K_{IC}^2}{E}$

Significance of the first equation (K_{IC})?:

K_{IC} combines the flaw size and the applied stress into a global parameter. It is now the engineering parameter for mechanical design.

Significance of the second equation ($2\gamma_F$)?:

$2\gamma_F$ is the work of fracture, that is how much energy is expended to grow the crack by one unit area.

We must now consider the atomistic events at the crack tip to compute a value for $2\gamma_F$ from a materials science point of view. This is the purpose of today's lecture.

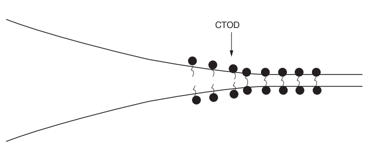
Why linear elastic fracture mechanics works?

It works because the mechanical energy which provides the driving force for crack propagation is from load-displacement and **elastic energy** considerations, regardless of the local events at the crack tip. The concept is viable even if the local deformation at the crack tip which leads to fracture occurs over a length scale that is far smaller than the size of the body which is providing the elastic energy.

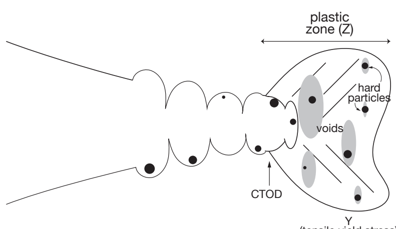
Examples of different kinds of fracture processes at the crack tip:

We shall discuss two types of "fracture mechanisms" at the crack tip - for brittle fracture and for small scale deformation and fracture at the crack tip, as in polymers and high strength metals.

Fracture with and without small scale yielding at the crack tip



Brittle Fracture



SemiBrittle of fracture with small scale yielding at the crack tip.

Fibril-fracture at a crack tip in a polymer.

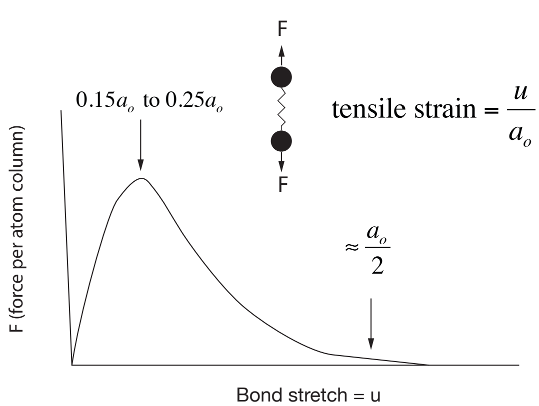
Criterion for fracture for the three cases just above:

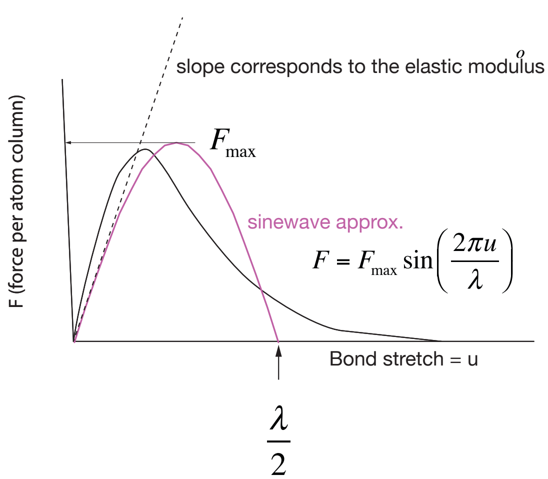
In brittle fracture the bond at the crack tip is stretched to a point that it breaks. The work fracture ($2\gamma_F$) is the mechanical work done to break one bond, multiplied by the number for bonds per unit area of the crack extension surface.

In Semi-Brittle fracture there is local yielding and plastic tearing for moving the crack forward. The criteria for fracture would be (a) that the local stress exceeds the yield stress, and (b) locally the plastic yielding stretches the material to the point that it separates. The work of fracture is the product of the local force (which is related to the yield stress) and the displacement that must be accommodated to tear/separate the material. The work of fracture would be equal to (yield stress)*(CTOD-crack tip opening displacement), which already has units of J m^{-2}.

In a polymer fracture progresses by the rupture of ligaments at the crack tip. The work to break one ligament is the force required to stretch it and the displacement required to break the ligament. Multiplying this quantity by the number of fibrils per unit area will give the value for $2\gamma_F$.

Case I: Ideal fracture: crack advances by breaking the bonds





Here a_o is the interatomic spacing ($=\Omega^{1/3}$ where Ω is the volume per atom - or molecule - calculated from molecular weight, density and the Avogadro's number).

Assumption:

What are the three elements of the curve?

(i)The initial slope of the sine wave ($u \to 0$) is related to the elastic modulus.

(ii)F_{max} is related to the rupture load or the fracture stress.

(iii)Work done to rupture the bond is the $\int_0^{\lambda/2} F\,du$. Therefore the work of fracture is the value of this integral multiplied by the number of bonds per unit area = $\frac{1}{\Omega^{2/3}}$.

Let us now combine the three elements to check whether or not $\frac{K_{IC}^2}{E} = 2\gamma_F$ holds.

THE APPROACH

We wish to express the result in terms of E, the elastic modulus, and K_{IC} the stress intensity factor.

•We calculate the fracture stress in terms of the elastic modulus from the sine-wave.

•Use the equations that describe the stress in front of the crack tip in terms of K_I in this way we relate the fracture criterion to the stress intensity factor.

•We calculate the work done for breaking bonds, which is again calculated from the sine-wave and thereby find the relationship between $\frac{K_{IC}^2}{E} = 2\gamma_F$.

The equation for the sine wave:

$$F = F_{\max} \sin(\frac{2\pi u}{\lambda}) \qquad (1)$$

Elastic modulus: It is related to the initial slope of the sine-wave.

$$\left(\frac{dF}{du}\right)_{u\to 0} = F_{\max}\frac{2\pi}{\lambda}\cos(\frac{2\pi u}{\lambda}) \quad \text{when u->0,} \quad \left(\frac{dF}{du}\right)_{u\to 0} = F_{\max}\frac{2\pi}{\lambda}$$

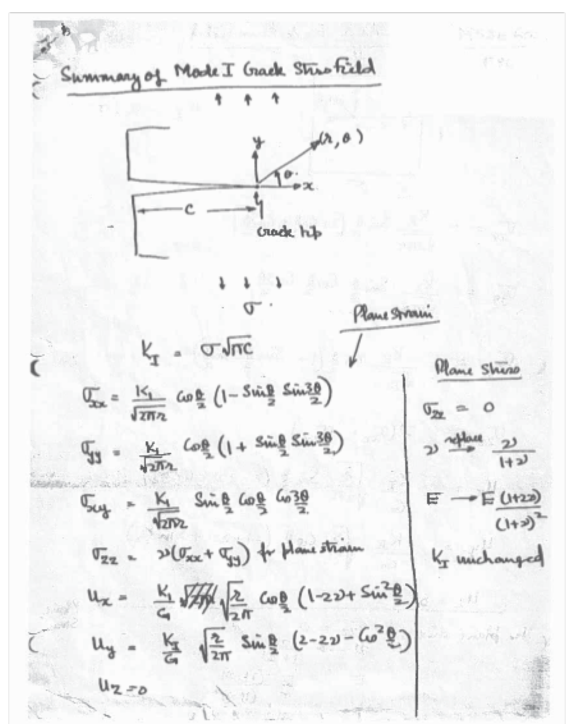
(2)

Writing in terms of stress and strain:

$$\sigma_{yy} = \frac{F}{\Omega^{2/3}};\ \varepsilon = \frac{u}{\Omega^{1/3}}$$

Fracture criterion: the first bond just ahead of the crack tip should experience a tensile stress $\sigma^*_{yy} = \frac{F_{\max}}{\Omega^{2/3}}$ (3)

Now to relate F_{max} to the elastic modulus we use equation (2):



$$\left(\frac{dF/\Omega^{2/3}}{du/\Omega^{1/3}}\right)\frac{\Omega^{2/3}}{\Omega^{1/3}}_{u\to 0} = F_{max}\frac{2\pi}{\lambda}$$
$$\left(\frac{d\sigma}{d\varepsilon}\right)_{u\to 0} = F_{max}\frac{2\pi}{\lambda}\frac{1}{\Omega^{1/3}} = E \tag{4}$$

$$\sigma_{yy}^{*} = \frac{F_{max}}{\Omega^{2/3}} = \frac{1}{\Omega^{2/3}}\frac{E\Omega^{1/3}\lambda}{2\pi} \tag{5}$$

we had assumed that $\lambda = \Omega^{1/3}$, substituting above we get

$$\sigma_{yy}^{*} = \frac{E}{2\pi} \tag{6}$$

Strain to fracture = $\varepsilon_f = \dfrac{1}{2\pi}$ which is about 15% (7)

From the results in hand written notes the local stress is related to the stress intensity factor. Enforcing the critical condition for crack propagation that the tensile load at the first bond nearest to the crack tip reaches the value given in Eq. (6) - it is easier to think in terms of the strain to fracture as given in Eq. (7).

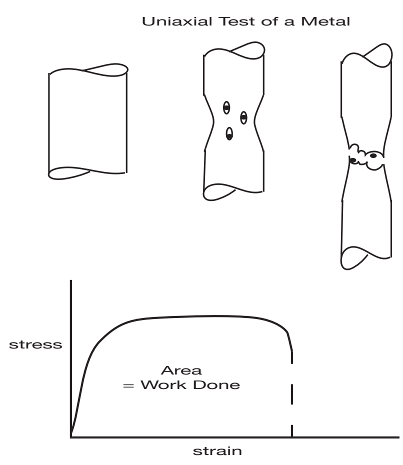
$$\sigma_{yy}^{*}(\theta = 0, r = \Omega^{1/3}) = \frac{K_{IC}}{\sqrt{2\pi\Omega^{1/3}}} = \varepsilon_f E \tag{8}$$

$$\frac{K_{IC}^2}{E} = 2\gamma_F = 2\pi\Omega^{1/3}\varepsilon_f^2 E \tag{9}$$

I have related the fracture toughness to the work of fracture in terms of the local parameters that define fracture which is just the s$\dfrac{1}{\Omega^{2/3}}$train to fracture equal to $\dfrac{1}{2\pi}$.

Home work problem asks you to calculate the value on the right hand side from the area under the half sine wave, which when divided by (number of bonds per unit area) gives an expression for $2\gamma_F$. This result can now be compared with Eq. (9) which is derived from the global analysis that $\dfrac{K_{IC}^2}{E} = 2\gamma_F$.

Case II: Semi-brittle fracture



Work done in tensile fracture is related to the yield strength and ductility as in a simple uniaxial test:

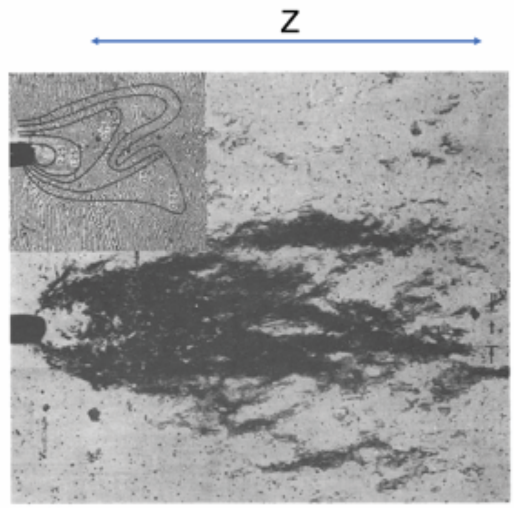


FIG. 7. Interference pattern with isostrain contours (top left corner) and the corresponding plastic zone revealed by etching, both for Sample S-56 (t = 0.017 in., T/Y = 0.81). ×17.5.

$$2\gamma_F = CTOD * Y$$

$$2\gamma_F = \frac{K_{IC}^2}{E} = CTOD * Y$$

$$CTOD = \frac{K_{IC}^2}{YE}$$

Case III: Fibril fracture for crack growth in polymers

Not dealt with here because the crack tip analysis is much more complex since the stress and strain are not uniform in the deformation zone.

Fibril-fracture at a crack tip in a polymer.