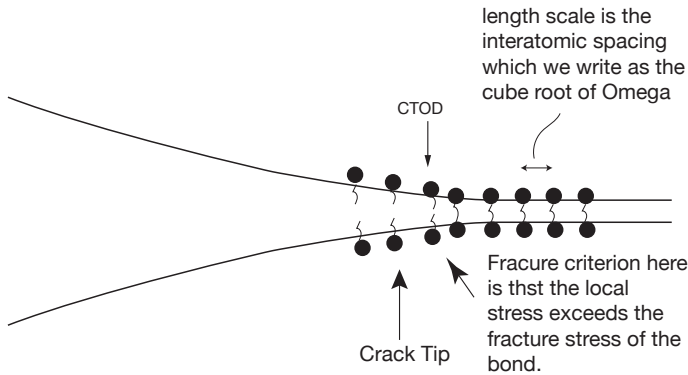


04/01/2021

HW17: Fracture Toughness of Glass from consideration of the stresses and displacements at the crack tip.



Summary of Mode I Crack Stress Field

Plane Strain

$$K_I = \sigma \sqrt{\pi c}$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} (1 - \sin\frac{\theta}{2} \sin\frac{3\theta}{2})$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} (1 + \sin\frac{\theta}{2} \sin\frac{3\theta}{2})$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \cos\frac{\theta}{2} \cos\frac{3\theta}{2}$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \text{ for Plane strain}$$

$$u_x = \frac{K_I}{G} \sqrt{\frac{2}{\pi r}} \cos\frac{\theta}{2} (1 - 2\nu + \sin^2\frac{\theta}{2})$$

$$u_y = \frac{K_I}{G} \sqrt{\frac{2}{\pi r}} \sin\frac{\theta}{2} (2 - 2\nu - \cos^2\frac{\theta}{2})$$

$$u_z = 0$$

Plane Stress

$$\sigma_{zz} = 0$$

$\nu \rightarrow \frac{2\nu}{1+\nu}$

$$E \rightarrow \frac{E(1+\nu)}{(1+\nu)^2}$$

K_I unchanged

In class (as seen in the notes from yesterday) we estimated the fracture toughness of glass assuming that the local tensile stress near the crack tip at a distance $\Omega^{1/3}$ from the crack tip is equal to the ideal tensile stress, which we had derived in the first part of the course (find it on the website, please). So the that **critierion for crack propagation** became

$$\sigma_{yy}(r = \Omega^{1/3}, \theta = 0) = \frac{E}{2\pi}$$

In this problem you are asked to use the displacement near the crack tip rather than the tensile stress as the fracture criterion according to

$$u_y(r = \Omega^{1/3}, \theta = 0) = \frac{\Omega^{1/3}}{2},$$

which the same criterion we had used for the derivation of the modulus etc. with the half-sine wave model.

Derive the equation for K_{IC} with this displacement criterion and compare it to the result derived in class, where the ideal tensile stress criterion was used as the crack propagation criteria.