

# Introduction to Mechanical Properties

## Length Scales

### •Why important?

Because they help to link features at the atomic level (for example the strength and stiffness of the bonds) to behavior which can be measured in the laboratory.

### Topic: Elastic stiffness of a spring, or a chain of molecules such as in DNA.

Note that in and in the DNA molecule the structure is defines by a "unit cell" which repeats itself.

Example: Mechanical Properties of a DNA

### •Elements of a Model?

Ingredients of the model: For example consider the elastic stiffness of a DNA molecule.

Structure: ordering of the pieces that make up the whole. a polymer, chain structure.

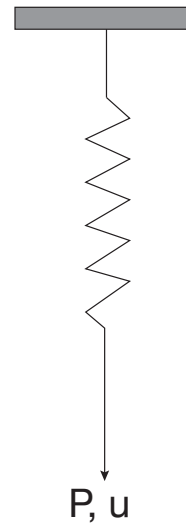
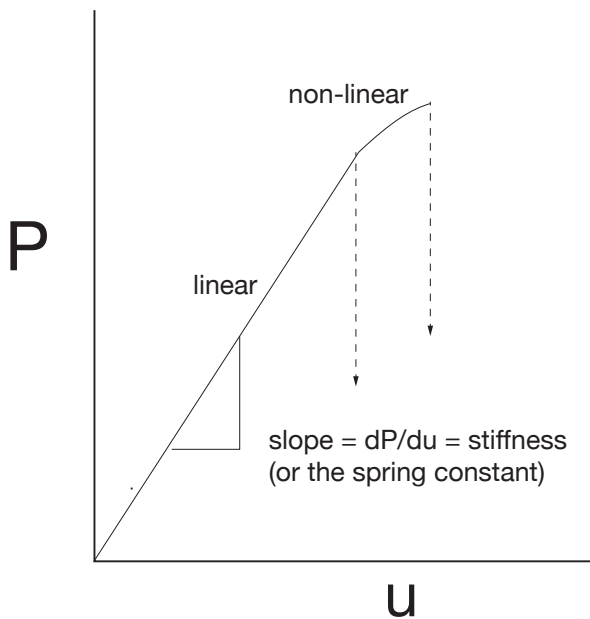
What are the different length scales? Length of the whole strand (100 $\mu$ m, monomers 3.4 nm, and chemical species 0.2 nm).

What may be the underlying physics for linking these widely spread length scales?

Bonding between atoms ---> bonds lead to description of a cluster of atoms ---? clusters form chains --> the DNA and a certain structure. The double helix.

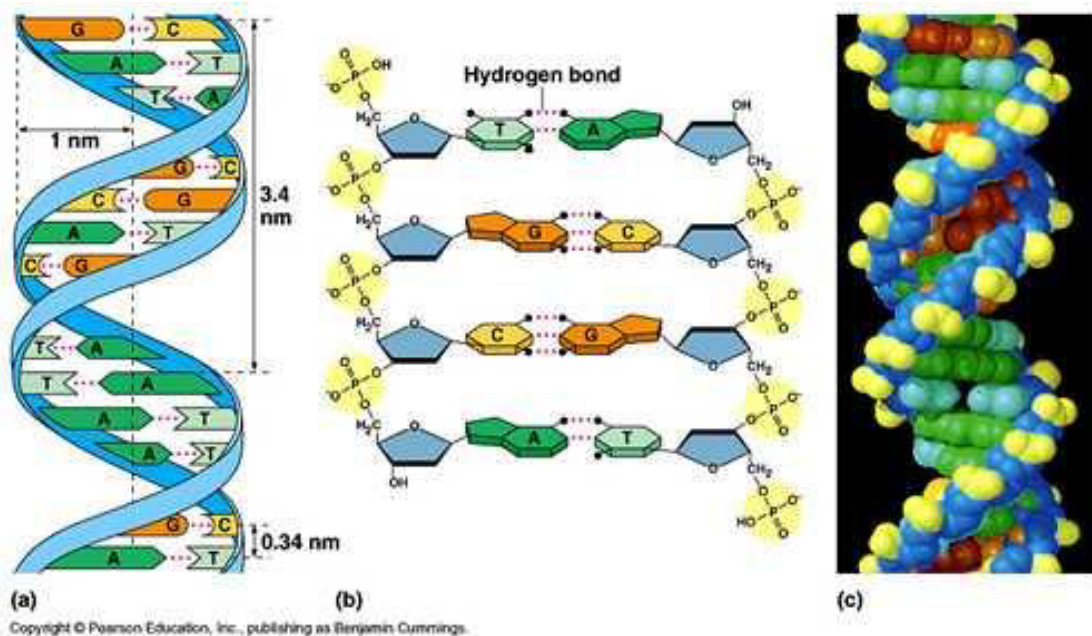
The double helix consists of repeating units so that the whole chain can be expressed as these units bonded in series.

The repeating unit has a certain length ( 3.4 nm). **We wish to connect the load displacement curve to the force/displacement behavior of one unit.**



The synergy between technology and science. For example we can now image atoms in the electron microscope. We can grab hold of the DNA chain with "optical tweezers".

That technology has enabled experiments to obtain a force displacement curve for a single molecule.



The idea is to divide the length of the specimen by the length on one unit to obtain the number of units in the chain that has been measured.

$$N = \frac{L_{\text{chain}}}{\text{length of one unit}} = \frac{L_{\text{chain}}}{\Omega^{1/3}}$$

Now we know the length of the chain (from optical microscopy) and the size of the unit from chemical structure of the DNA. Therefore we know the value for  $N$  in the chain.

Here the unit is a chain segment. Later, in more general scenarios, we like to consider the volume of one unit. We write this volume as  $\Omega \text{ m}^3$ . So for example the size of the unit is  $a$ , then  $a^3 = \Omega$ , therefore

$$a = \Omega^{1/3}$$

The property of the DNA would we wish to measure?

-Energy absorbed before the molecule breaks? (Always a limiting condition to a mechanism of deformation)

-Measure the elastic stiffness of the chain. ( $P, u$ ) behavior.

Each unit is in series with the other. And all units are experiencing the same load.

$$P = k_{\text{chain}} u$$

$$P = k_{\text{unit}} u_{\text{unit}}$$

$$u = N u_{\text{unit}}$$

$$\frac{P}{k_{\text{chain}}} = N \frac{P}{k_{\text{unit}}}$$

$$k_{\text{unit}} = N k_{\text{chain}}$$

Note that the above equation could have been written down from intuition and empirical knowledge:

(i) The stiffness of one bond would be greater than the stiffness of the chain

(ii) The result would be either to multiply the stiffness of the chain by the number of bonds in the chain or to divide by  $N$ .

(iii) Units check: on both sides of the equation are balanced. Yes!

01\_Practice HW (go to HW page)