#6: Principal Stresses and Strains

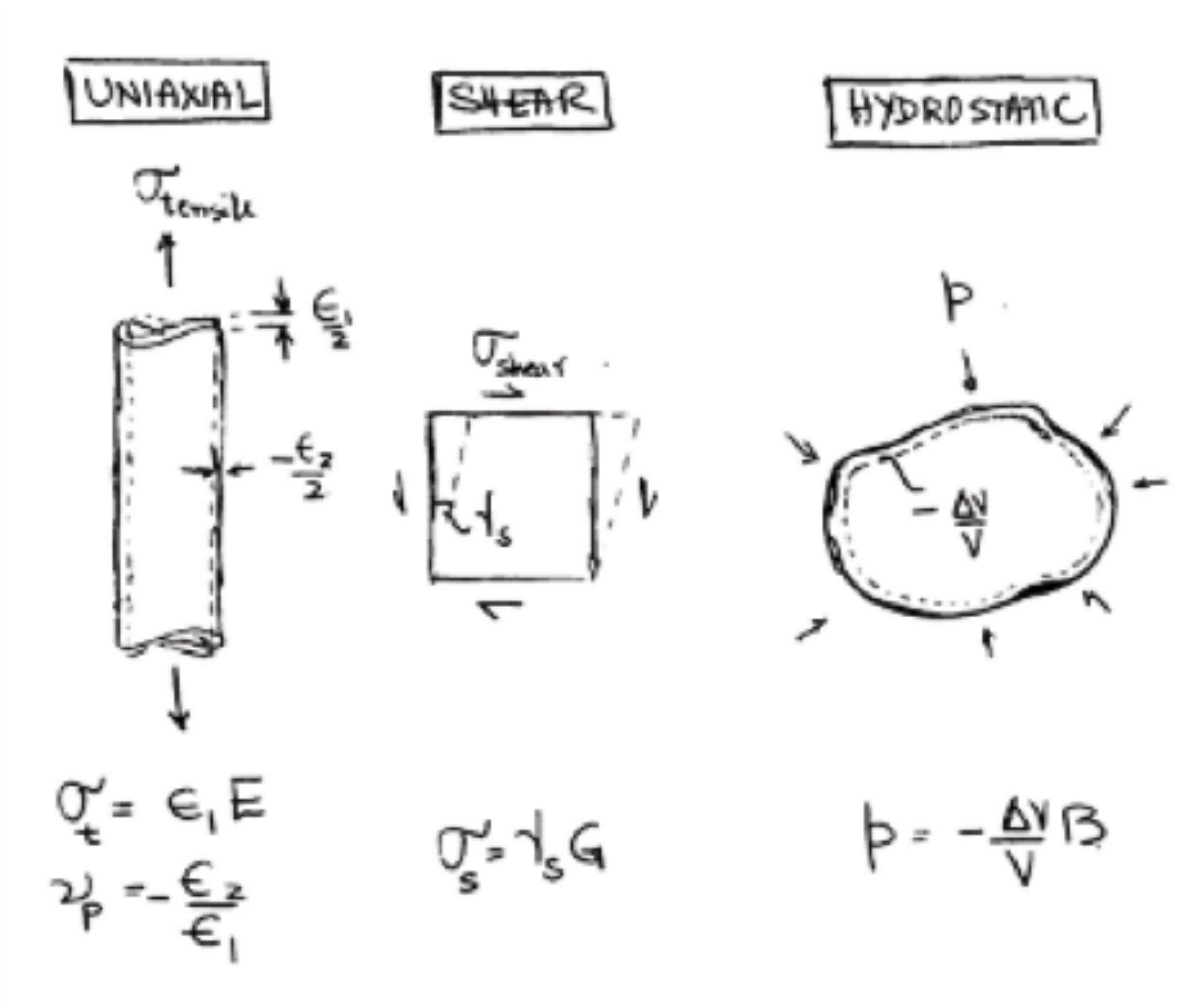
•Three values instead of six for the general case compensated by the loss of three degrees of freedom

•Significance of the sum and differences for the representation of hydrostatic and shear components of the stress state.

•Explanation of shape and volume change in a tensile and shear experiment.

•Multiaxial deformation.

•Number of independent elastic constants for a uniaxially aligned graphite fiber reinforced polymer composite (GFRP).



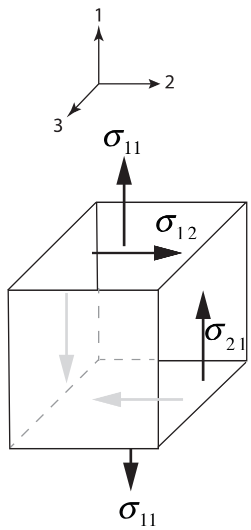
The General Stress Tensor

It is a second rank tensor because it requires two orthogonal vectors to describe it.

•One is the direction of force

•Is the normal to the plane to which the force is applied

$\sigma_{21} = \sigma_{12}$



There are six independent components of the stress tensor.

It is self evident these stress components (is magnitudes) will change if the cube in rotated in different ways.

The great news is the one can find a unique orientation where only the stress components acting normal to the faces are non-zero:

$\sigma_{11} \rightarrow \sigma_1$

σ_2

σ_3

Correspondingly we have three principal strains.

ε_1

ε_2

ε_3

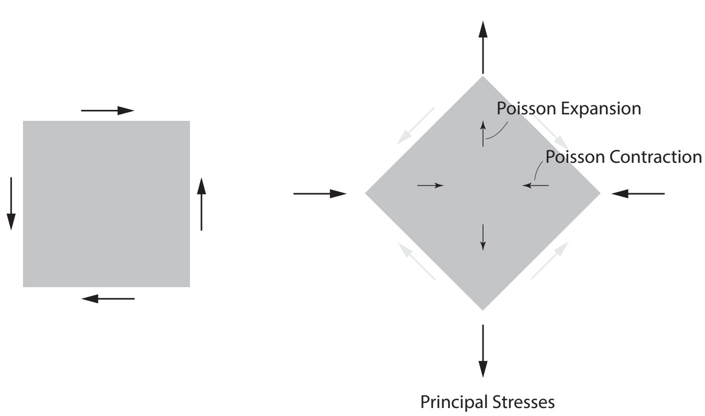
We have said that hydrostatic loading and shear loading are the two fundamentally independent ways of applying stress.

$$\sigma_H = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

•Note that hydrostatic stress is in three dimensions.

Shear stress depends on the difference between any two principal stresses.

•Shear deformation occurs in two dimensions, on a cross section with unequal principal stresses.



Now the principal stress components on the right are $+\sigma$ in the vertical, and $-\sigma$ in the horizontal direction.

Shear stress is the difference between the principal stresses, that is

Shear stress = $\sigma-(-\sigma)$ =(constant) 2σ

I will let you analyze what this constant should be.

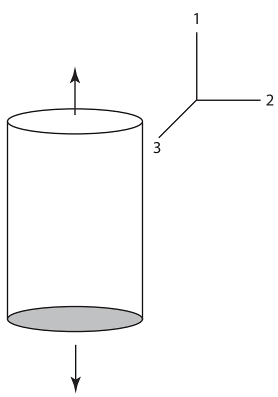
Any representation of principal stresses (three of them) can be decomposed into a hydrostatic component and a shear component.

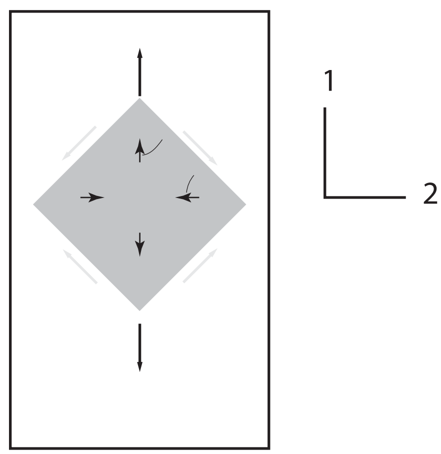
Let us consider a simple case of uniaxial elastic deformation,

$$\begin{matrix}\sigma_1 & & \\ & 0 & \\ & & 0\end{matrix} = \begin{matrix}\frac{\sigma_1}{3} & & \\ & \frac{\sigma_1}{3} & \\ & & \frac{\sigma_1}{3}\end{matrix} + \begin{matrix}\frac{2\sigma_1}{3} & & \\ & -\frac{\sigma_1}{3} & \\ & & -\frac{\sigma_1}{3}\end{matrix}$$

The first matrix is pure hydrostatic, and the second is pure shear (because the sum of the three principal stresses is equal to zero and the differences are non zero).

•There is no shear in the (2,3) plane, which is consistent with the principal stress matrix for a simple uniaxial experiment. the vo





Another problem,

$$B=\frac{E}{3(1-2\nu)}$$

$$\begin{matrix}\sigma & & \\ & \sigma & \\ & & \sigma\end{matrix} \quad \text{therefore } \sigma_H=\sigma$$

$$\begin{matrix}\sigma & & \\ & \sigma & \\ & & \sigma\end{matrix} = \begin{matrix}\sigma & & \\ & 0 & \\ & & 0\end{matrix} + \begin{matrix}0 & & \\ & \sigma & \\ & & 0\end{matrix} + \begin{matrix}0 & & \\ & 0 & \\ & & \sigma\end{matrix}$$

Thy hydrostatic state of stress has now been decomposed into three uniaxial states.

For example the first component will give a strain

$$\begin{matrix}\varepsilon_1 & & \\ & -\nu\varepsilon_1 & \\ & & -\nu\varepsilon_1\end{matrix} \quad \text{the volume change = the sum of the three principal strains, i.e. } \varepsilon_1(1-2\nu)$$