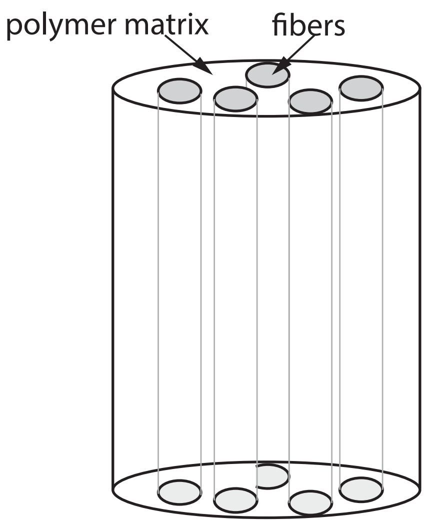
#7: Composites

Uniaxial Fiber Composites



Three steps in building models **(first you must describe the objective of the model)**

(i) Describe the microstructure with geometrical parameters

(ii) Give the handbook properties of the constituents

(iii) Develop a concept for solving the problem at hand

Model for the Elastic Modulus of the Composite in the Uniaxial Direction

Steps:

•We wish to describe the elastic modulus (Youngs modulus) of the composite in terms of its constituents.

• E_C relate it to the Youngs Modulus of the constituents (E_f, E_m)

•Geometry: orientation of the fibers, area fraction **or** the volume fraction.

Why is the volume fraction always equal to the area fraction?

Consider a thin slice through the composite. The thickness of the slice = δ , and we assert that the thickness is much smaller than the length of the microstructure (for example if it is a particulate composite, then the thick is much smaller than the size of the particles).

Volume of the particles in the slice = total area of the slice*area fraction of the particles*thickness of the slice

Total volume of the slide = total area of the slice*thickness of the slice

Volume fraction of the particle = area fraction of the particles in a random cross section. This relationship applies to any geometry of a composites.

To put it together (simple mechanics)

We enforce the condition that the uniaxial strain in the matrix is equal to the uniaxial strain in the fibers, and the are equal to the tensile strain experienced by the composite.

$$\frac{\sigma_f}{E_f} = \frac{\sigma_m}{E_m} = \frac{\sigma_C}{E_C}$$

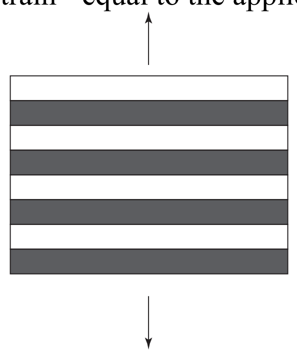
From equilibrium

$$\sigma_C = \sigma_f * v_f + \sigma_m * (1 - v_f)$$

$$E_C = v_f E_f + (1 - v_f) E_m \qquad (1)$$

Model for the Elastic Modulus of the Composite in the Transverse Direction

In this instance the constituents experience the same stress, which is equal to the applied stress (in the longitudinal direction the fibers and the matrix had the same strain - equal to the applied strain).



Here the fibers have a volume fraction of 50%

Now the total strain is the sum of the strain in the matrix and the strain in the fibers. Recalling the following relationship between strain, stress, and the E, the Young's Modulus,

$$\varepsilon = \frac{\sigma}{E},$$

and adding up the strains in each constituents (weighted by their volume fractions) to obtain the total strain, and then then equating the total strain to the applied stress divided by the effective Young's modulus of the composite in the transverse direction would lead to

$$\frac{1}{E_{comp}} = \frac{v_f}{E_{fiber}} + \frac{1 - v_f}{E_{polymer}} \qquad (2)$$

Can you derive this equation?

Hashin Bounds

Equations (1) and (2) above form the upper bound and the lower bound for the elastic modulus of the composite. Composites with another microstructure for example will fall within these bounds. The bounds are shown schematically in the figure on the right.