# #18: Fracture: Crack-tip criterion of fracture

Overview

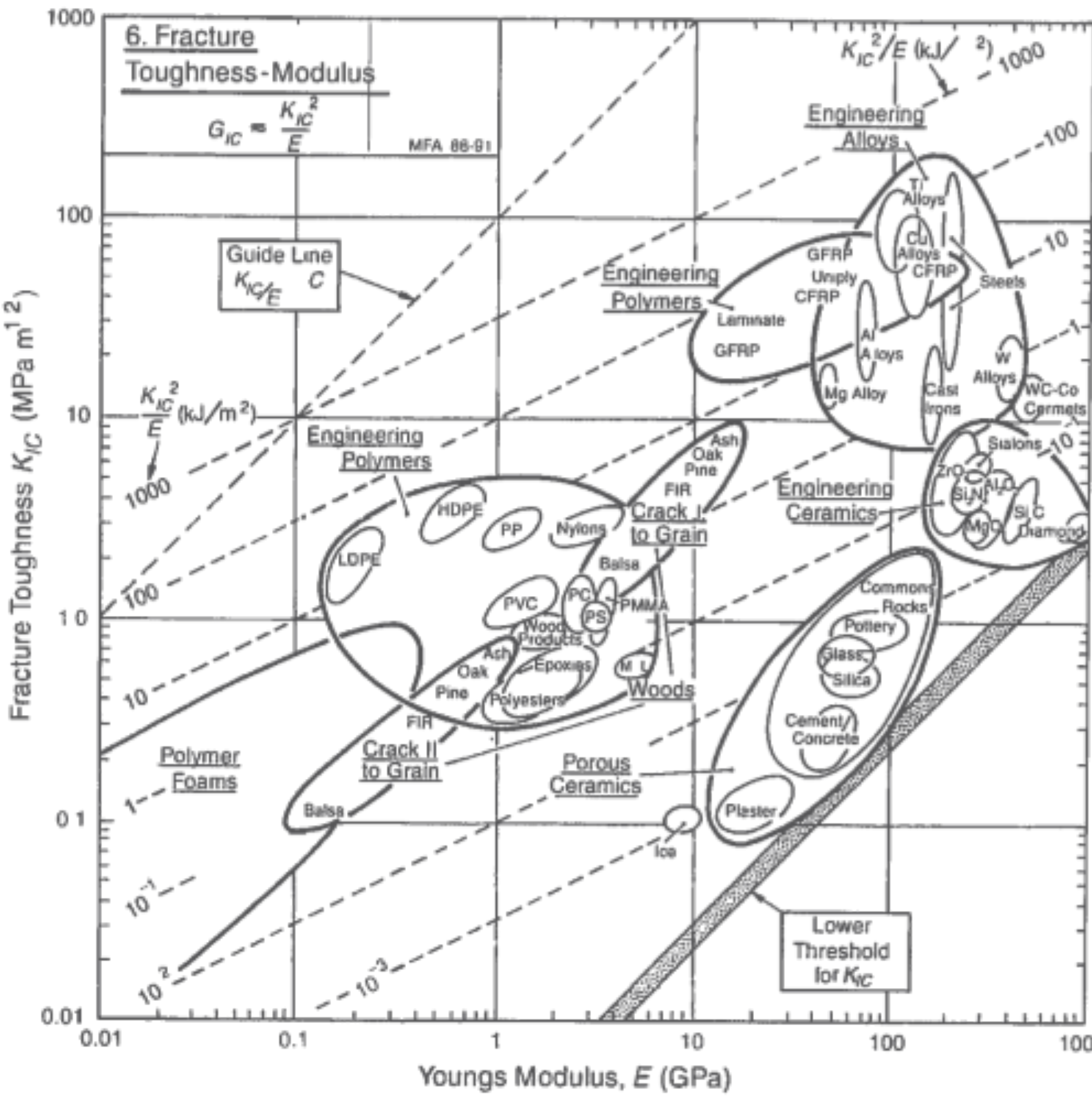
•Unitl now we have only considered global **energy balance equations** to derive the following result (see the last section of the last lecture), that is

$$2\gamma_F = \frac{K_{IC}^2}{E} \qquad (1)$$

where all three are material parameters.

•Among them $2\gamma_F$ is clearly related to materials science since it describes the work done to increase the surface area of the crack by one unit, or 1 m$^2$. The experimental values for $2\gamma_F$ vary greatly as shown in the map on the right. Here the dotted lines with a slope of 1:2, that is, when $K_{IC}$ increases by one order of magnitude then $E$ increases by two orders of magnitude, show that materials lying on one line will have the same magnitude of the work of fracture. For example polymers have similar values for $2\gamma_F$ as Aluminum alloys.

•However, note that the fracture toughness of polymers is about 1 MPa m$^{0.5}$, whereas aluminum alloys have a toughness of 10 MPa m$^{0.5}$.




•The above result can be explained by the difference in the Youngs modulus, which is higher for the metal by two orders of magnitude. Since $K_{IC} \propto \sqrt{E}$, two orders of magnitude increase in $E$ should indeed lead to one order of magnitude change in $K_{IC}$.

Therefore, we conclude that we need to understand why the work of fracture for polymers is so much higher, in order to understand the plots given in the map. The work of fracture is a physical process by which the material comes apart at the crack tip. (The great value of the energy calculation has been that it can lump the local work of fracture into one parameter, and measure it from global energy consideration, which we have done quite extensively. **(Note that these analyses apply only if the deformation away from the crack tip is elastic, so that plastic deformation if any is confined close to the crack tip).**

In summary we attempt to calculate the work done at the crack tip assuming that the overall deformation in the specimen is elastic, since the change in the elastic energy was used to obtain the equation $2\gamma_F = \frac{K_{IC}^2}{E}$.

# Different Types of Fracture Mechanisms as the Crack tip

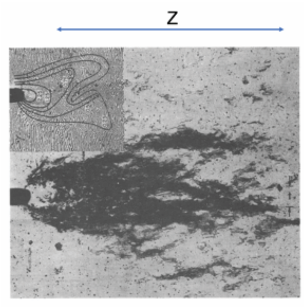
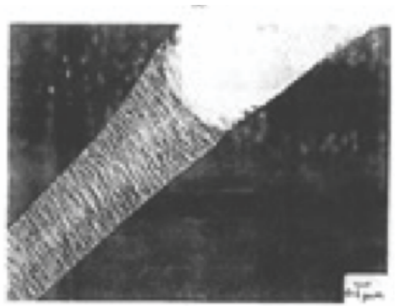


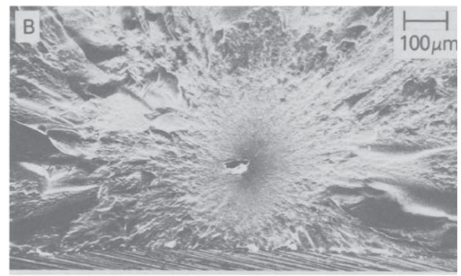

FIG. 7. Interference pattern with isostrain contours (top left corner) and the corresponding plastic zone revealed by etching, both for Sample S-56 (t = 0.017 in., T/Y = 0.81). ×17.5.

$2\gamma_F = CTOD * Y$

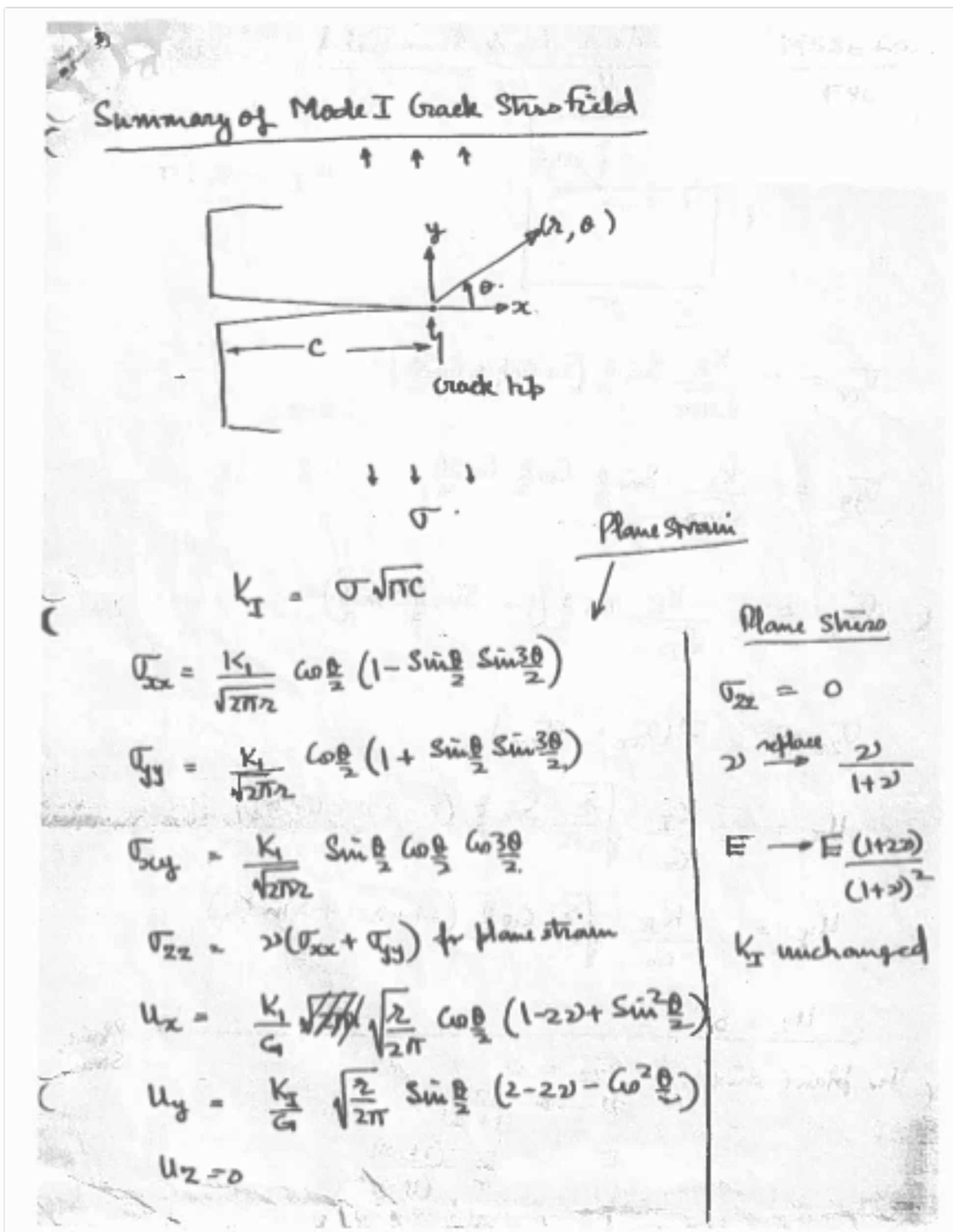
$2\gamma_F = \frac{K_{IC}^2}{E} = CTOD * Y$

$CTOD = \frac{K_{IC}^2}{YE}$






To analyze these fracture mechanisms (the goal is to relate them to $2\gamma_F$) we must have expression for stresses and displacements at the crack tip. These are given by the following equation for pure elastic deformation, on the right hand side.



Summary of Mode I Crack Stress field

$(r, \theta)$

Plane Strain

Plane Stress

$\sigma$.

$K_I = \sigma\sqrt{\pi c}$

$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$

$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$

$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$

$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$ for plane strain

$u_x = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(1 - 2\nu + \sin^2\frac{\theta}{2}\right)$

$u_y = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(2 - 2\nu - \cos^2\frac{\theta}{2}\right)$

$u_z = 0$

$\sigma_{zz} = 0$

$\nu \xrightarrow{\text{replace}} \frac{\nu}{1+\nu}$

$E \rightarrow E\frac{(1+2\nu)}{(1+\nu)^2}$

$K_I$ unchanged

CONSIDER THE CASE OF BRITTLE FRACTURE LIKE IN SILICA GLASS

$K_{IC}$ 0.6-0.8 MPa $m^{0.5}$

E 76 GPa

Putting in these values

shows the work of fracture in terms of the measured values of fracture toughness and the elastic modulus.

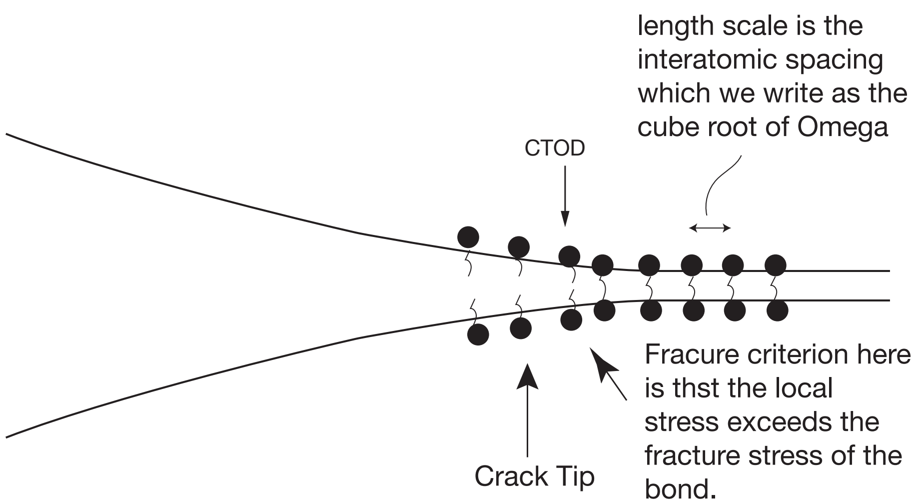
We measure the surface energy from fracture experiments and compare them with the measurement from chemistry and physics in the form of the surface energy.

As done below we obtain that $\gamma_F = 2.37\ \mathrm{J\ m^{-2}}$

But in physics (for example by measurements of surface tension we find that $\gamma_S \approx 1\ \mathrm{J\ m^{-2}}$. Therefore we are led to conclude that the fracture is not is not ideally brittle in glass but does involve some deformation at the crack tip.

| | | |
|---|---|---|
| K_IC | 6.00E-01 Mpa m^0.5 | |
| | 6.00E+05 Pa m^0.5 | |
| E | 7.60E+01 Gpa | |
| | 7.60E+10 Pa m^0.5 | |
| 2Gamma_F | 4.74E+00 J m^-2 | |
| Gamma_F | 2.37E+00 J m^-2 | |
| surface energy | 1.00E+00 J m^-2 | nothing to do with fracture |

# Model for brittle fracture by the scission of bonds at the crack tip




In words, the tensile stress at the crack tip at distance of interatomic spacing ($\Omega^{1/3}$ ) is equal to the fracture strength of the bond.

What is the fracture strength of the bond?

As derived during consideration of elastic deformation (for to Elasticity page and look for Bond Energy)

$$\sigma_F^{ideal} = \frac{E}{2\pi}$$

Force per bond at fracture = $F * \Omega^{2/3}$ , the latter being the area occupied by one bond .

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2})$$

so now we say that

when $r = \Omega^{1/3}$ and $\theta = 0$, the local stress = $\frac{E}{2\pi}$ (this is the fracture criterion). Enforcing this condition we get an expression for $K_{IC}$ which is $\frac{E}{2\pi} = \frac{K_{IC}}{\sqrt{2\pi\Omega^{1/3}}} * 1$

$$K_{IC} = \frac{E\sqrt{\Omega^{1/3}}}{\sqrt{2\pi}}$$ (grand result)

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Method I: Use the bond energy calculated in early lecture

E                              75GPa
                  75000000000Pa
Mol wt                         60g/mol
Density                      2.65g/cm^3
Molar vol         22.64150943cm^3
                  2.26415E-05m^3
N_A                  6.03E+23#/mol
Omega                3.75E-29m^3
Omega^1/3            3.35E-10m^3

K_IC                 5.48E+05Pa m^1/2            5.48E-01MPa m^1/2
Handbook     0.6 to 0.8      MPa m^1/2
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$$K_{IC} = \frac{E\sqrt{\Omega^{1/3}}}{\sqrt{2\pi}}$$

Therefore, the calculated value from the model comes close to the experimental value of the fracture toughness.

## Model for semi-brittle fracture where there is deformation and damage at the crack tip which extends the crack when this damage reached a critical value.

Note the elements of the fracture mechanism at the crack tip. They are,

- The local stress is equal to the yield stress of the metal
- The plastic zone size is the physical distance of the damage zone in front of the crack tip
- The crack tip opening displacement (CTOD) is the local "tearing apart" that is when the damage causes separation and the crack moves forward.

We must now consider the above parameters and build them into a model that related them to the measured value of the fracture toughness.