Take Home Exam04D: Strain Rate Equation

Assigned: 04/26/2022 (Tueday)

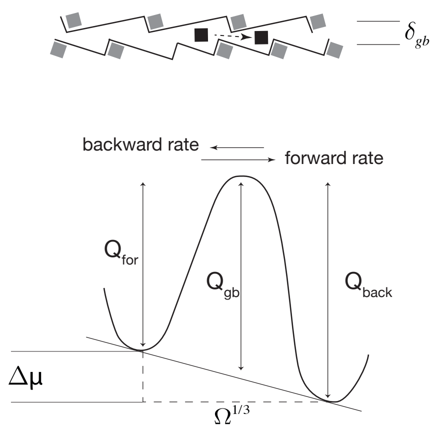
Due (as pdf by email) 05/01/2022 (Sunday-before 5 PM)

Please send your submission via email starting with HWExam04D in the subject line.

This HW will be expanded after lecture on Thursday, and together these problems will be due on Sunday, May 01.

04D.1

In this problem you are asked to derive an equation for the *jump frequency of atom* (as in solid state diffusion) down a gradient of the chemical potential, along a grain boundary, as sketched below



Note that the gradient of the chemical potential, that is, the driving force for diffusion flux, is equal to

$$\frac{d\mu}{dx} = \frac{\Delta\mu}{\Omega^{1/3}}$$

Note that the potential barrier creates a larger probability of forward jumps that backward jumps. The net forward jumps are then given by

$$\Gamma_{forward} - \Gamma_{backward} = \frac{\nu a^2}{6}\left(e^{-\frac{Q_{forward}}{RT}} - e^{-\frac{Q_{backward}}{RT}} \right)$$

Show that, if $\Delta\mu_{per\,mol} \ll RT$, the forward diffusion current will be given by

Net forward diffusional current = $D_{gb}\dfrac{\Delta\mu_{per\,mol}}{RT}$

where $D_{gb} = \dfrac{\nu a^2}{6} e^{-\frac{Q_{gb}}{RT}}$

please recall that in class we derived that $\Delta\mu = \sigma\Omega$ on a per atom basis. The condition on the left on a per atom basis will be $\Delta\mu \ll k_B T$, since $R = k_B N_A$

(Note that the net forward diffusion is linearly related to the driving force if $\Delta\mu_{per\,mol} \ll RT$)

04D.2

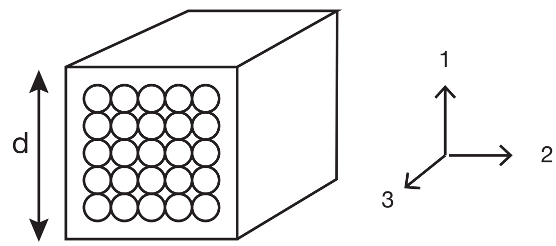
In yesterday's lecture we derived the following equation for the strain rate under *uniaxial stress*, σ

$$\dot{\varepsilon}_{tot} = 8\frac{\sigma\Omega}{k_B T}\left(\frac{D_V}{d^2} + \frac{\delta_{gb} D_B}{d^3}\right)$$

$$= 8\frac{\sigma\Omega}{k_B T}\frac{D_V}{d^2}\left(1 + \frac{\delta_{gb} D_B}{d D_V}\right) \qquad \text{(G)}$$

Principal stresses

	σ_1	σ_2	σ_3
Case I	$+\sigma$	0	0
Case II	$+\sigma$	$-\sigma$	0



The diagam on the right shows the stress state for the simple uniaxial stress (Case I), and for the biaxial stress state (Case II).

Derive the equivalent of Eq. (G) for Case II.

Hint: you do not need to go through the entire derivation. Think about what is different, and obtain the result from simple substitution into Eq. (G)

04D.3

Experiments are carried out to validate the grain size dependence of the strain rate predicted by Eq. (G), by keeping T and stress constant and varying the grain size. The data are plotted on a log-log scale with strain rate as the y-axis and the grain size as the x-axis. In this graph draw the lines for data expected for volume diffusion dominated and grain boundary dominated grain size dependence. The lines should have the expected slope for each case. Assume that the range covers a range that includes both d^2 and d^3 grain size dependence of the strain rate.