01C_The Mohr Circle

Topics:

•The Principal Stresses, again
•Using the Mohr Circle for finding the shear stress

Principal Stresses

•The Cartesian axes can be rotated into principal axes so that a general stress tensor becomes "diagonalized", that is only the terms along the body diagonal are non zero. We write them as $\sigma_1, \sigma_2, \sigma_3$.

•If all principal stresses are equal to one another then the "stress state" is "hydrostatic". Indeed the sum of the principal stresses divided by three gives the hydrostatic component of the stress state.

•Unequal principal stresses mean there is a non-zero shear stress. Shear stress act on a plane, i. e. a (1,2), (2,3) or the (1,3) plane.

01C.1 Why can the shear stress express in two dimensions (while hydrostatic stress required three dimensional description)?

•The Mohr Circle enables a graphic determination of the shear stress in two dimensions.

Construction of the Mohr Circle (with examples)

•Mohr Circle works in two dimensions, that is, in one plane of the Cartesian coordinates. It is most useful to obtain the value of the shear stress in anyone of the three Cartesian planes.

Let us consider the shear stress in the (1,2) plane, as in the following stress tensor

Since we consider only the (1,2) plane we set $\sigma_3 = 0$, so that $\begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & 0 \end{bmatrix}$

But similarly we can consider $\begin{bmatrix} \sigma_1 & & \\ & 0 & \\ & & \sigma_3 \end{bmatrix}$ for the (1,3) or, $\begin{bmatrix} 0 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix}$ for the (2,3) plane.

Note: that the stress tensor can be split up into two parts: one is pure hydrostatic and the other pure shear

$$\begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & 0 \end{bmatrix} = \begin{bmatrix} \frac{\sigma_1+\sigma_2}{2} & & \\ & \frac{\sigma_1+\sigma_2}{2} & \\ & & 0 \end{bmatrix} + \begin{bmatrix} \frac{\sigma_1-\sigma_2}{2} & & \\ & -\frac{\sigma_1-\sigma_2}{2} & \\ & & 0 \end{bmatrix}$$

where the second term on the right expresses pure shear since the sum of the principal stresses is equal to zero. The shear stress is therefore given by the difference in the two principal stresses divided by 2,

Therefore,

$$\sigma_S = \frac{\frac{\sigma_1-\sigma_2}{2} - \left(-\frac{\sigma_1-\sigma_2}{2}\right)}{2} = \frac{\sigma_1-\sigma_2}{2} \qquad \text{(01C.1)}$$

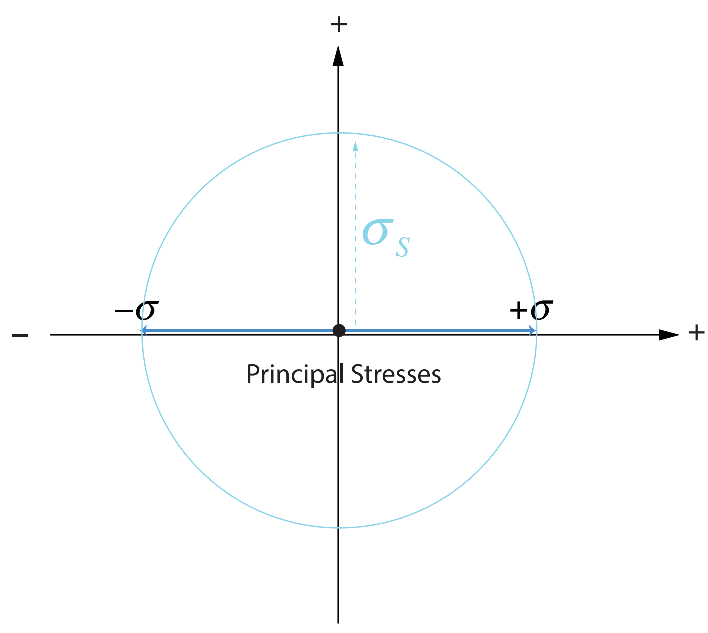
The equations in (01C.1) can now be represented graphically as follows,

Represent the principal axes along the horizontal axis. The mean position gives the hydrostatic component and the radius of the circle encompassing the two principal stresses gives the shear component.

•Let us consider the (1,2) plane. The horizontal axis represents the principal stress components. They can be positive or negative.

Pure Shear

$$\begin{bmatrix} +\sigma & \\ & -\sigma \end{bmatrix}$$



Mixed Shear and Hydrostatic

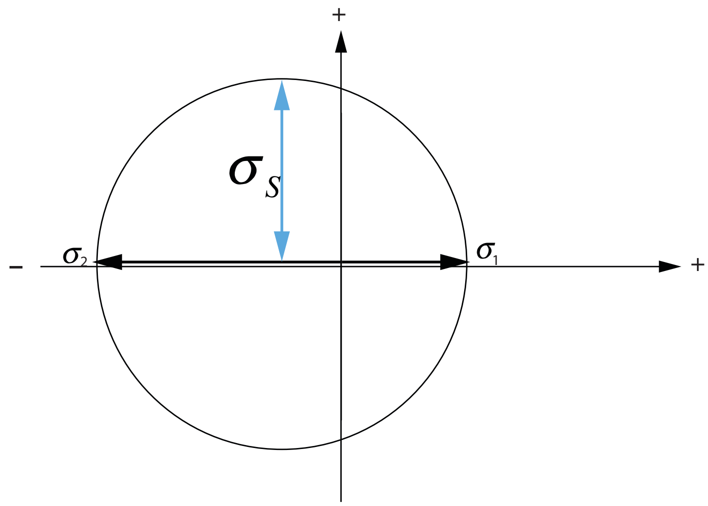
$$\begin{bmatrix} \sigma_1 & \\ & \sigma_2 \end{bmatrix}$$

•The vertical axis gives the shear stress. The shear stress iequal to the difference between the principal stresses divided by two, that is,

$$\sigma_S = \frac{\sigma_1 - (-\sigma_2)}{2}$$

The radius of the Mohr Circle drawn as shown below is then equal to the shear stress.

Draw a circle that touches the tips of the two vectors for the principal stresses. The radius of this circle is the shear stress, as shown on the left.



01C.2 Show the magnitudes of the hydrostatic and shear stresses in the Mohr Circle for the following cases (using arbitrary units)

(i) both principal stresses are equal to 5 units

(ii) one of them is +2, and the other is +10

(iii) one of them is -2 and the other is -10