Lect # 1

- Elasticity
  - Stress
  - Strain
  - Stored energy

- Geometry of Deformation
  - uniaxial 
  - cantilever beam.

- Uniaxial Defm (Physics)
  - Elastic Modulus
  - Strain ⇔ Stored Energy

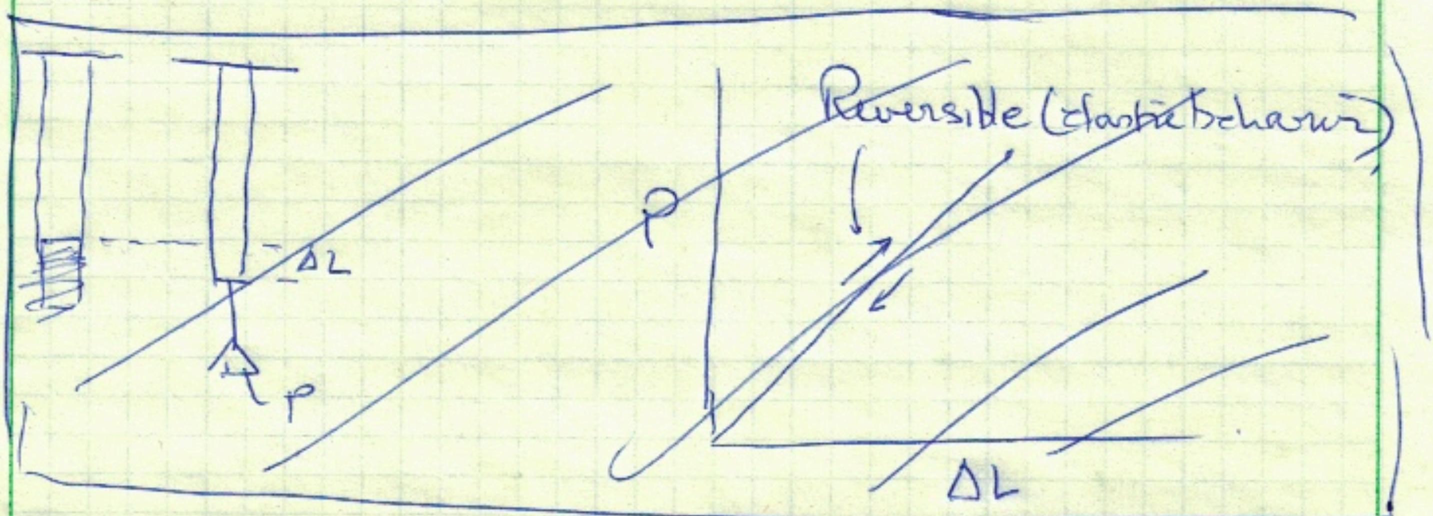
    ↕

    Energy of formation

- Cantilever Beam (Design)
  - atomic force microscope (DNA)
  - Diving Board

KTCH 1364

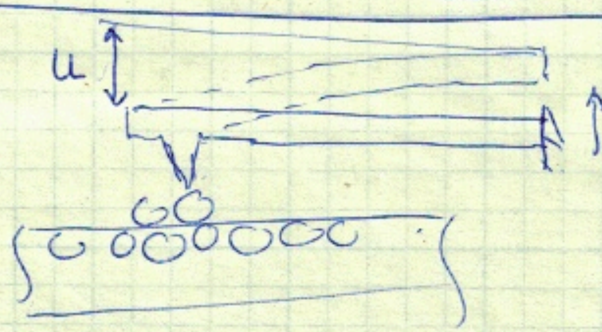
Elasticity



Design

AFM

Atomic Length Scale

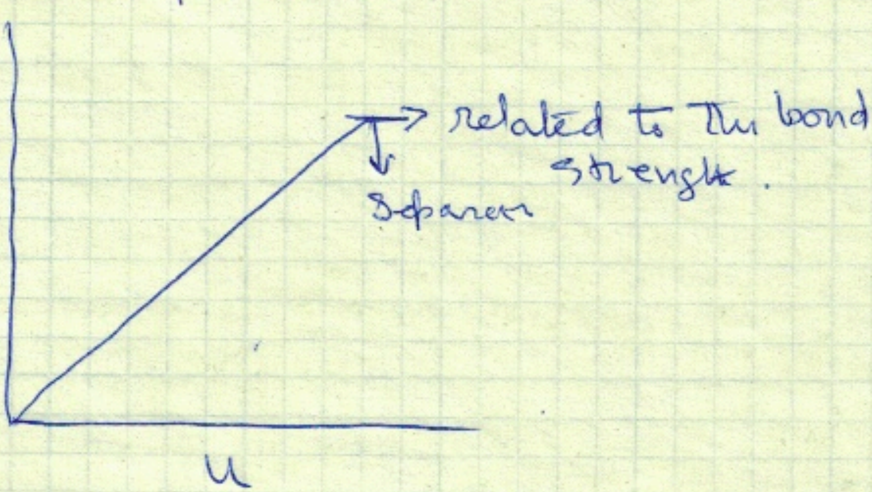


k = Bending stiffness of the beam

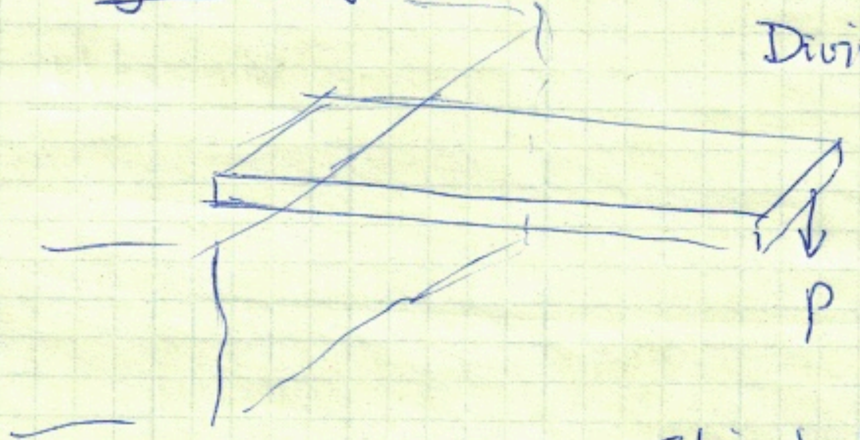
$$k = \frac{P}{u}$$

Deformation is complex

$$ku = P$$

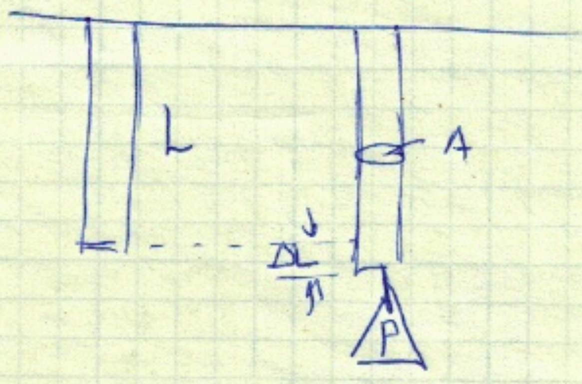


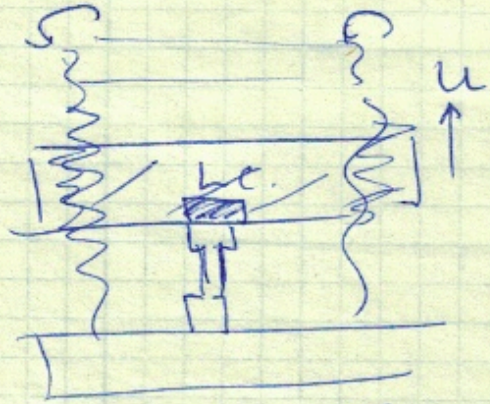
Physical Length scale

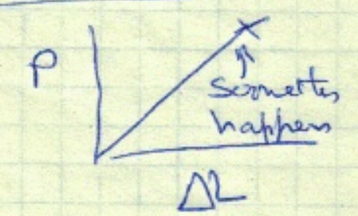


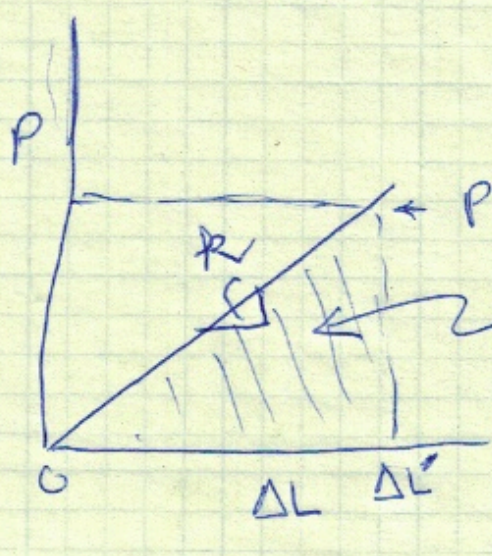
Springback (stored energy)

Consider a simple geometry









Stored elastic energy

$$U'_E = \int_0^{\Delta L'} P\, d(\Delta L) = \frac{1}{2} P' \Delta L'$$

Specimen Volume $= LA.$

$$U_E = \frac{U_E'}{LA} = \frac{1}{2}\frac{P'\Delta L'}{A\ L}$$

(energy per unit volume)

Scalar.
(universal quantity)

$$= \frac{1}{2}\left(\frac{P'}{A}\right)\left(\frac{\Delta L'}{L}\right)$$

$$= \frac{1}{2}\sigma\ \epsilon$$

strain (dimension less)

Force/unit area
$N/m^2 \equiv Pa$

k = stiffness.

$$k = \frac{P}{\Delta L} = \frac{P}{A}\cdot A \cdot \frac{1}{\frac{\Delta L}{L}} \times \frac{1}{L}$$

$\frac{N}{m}$

$$= \underbrace{\frac{\sigma}{\epsilon}}\cdot\frac{A}{l}$$

specific geometry

$$= E \times \boxed{\frac{A}{l}}$$

length

$Pa = \frac{N}{m^2}$

general

$$U_E = \frac{1}{2}\epsilon\sigma$$

$$= \frac{1}{2}\frac{\sigma^2}{E} = \frac{1}{2}E\epsilon^2$$

$E = 100\ GPa \qquad \epsilon = 0.01$

$$U_E\ (J/m^3) = \frac{100\times10^9}{2}\times10^{-2} = 5\times10^8\ J/m^3$$

Consider ZrO_2.

$$\text{Molar Volume} = \frac{\text{Mol. Wt (gm)}}{\rho \text{ density } (gm/cm^3)} = \frac{123 \text{ g/mol}}{5.68}$$

=

$$U_E = \frac{1}{2} E \epsilon^2 \quad \text{(energy/unit volume)}$$

$$U_E \; \frac{1}{2} E \epsilon^2 \times \cancel{\frac{1}{\frac{\text{Mol wt (g)}}{\rho (g/cm^3)}}} \; \frac{\text{Mol wt (g)}}{\rho (g/cm^3)} \times \underbrace{10^{-6}}_{cm^3 \to m^3} \; J/mol.$$

Energy per molecule

$$U_{E \,(\text{per molecule})} = \frac{10^{-6}}{2} \cdot E \epsilon^2 \frac{\text{Mol wt (g)}}{\rho (g/cm^3)} \times \frac{1}{N_A} \; J$$

Avogadro No.

$$1 \, eV = 1.6 \times 10^{-19} J$$

$$1 J = \frac{10^{19}}{1.6} eV$$

$$U_{E \,(eV)} = \frac{10^{-6}}{2} \times E \epsilon^2 \frac{\text{Mol wt (g)}}{\rho (g/cm^3)} \times \frac{1}{1.6} \times \frac{10^{19}}{6.02 \times 10^{23}}$$

$$= \frac{1}{2 \times 1.6 \times 6.02} \times 10^{-6+19-23} \times E \epsilon^2 \frac{\text{Mol wt (g)}}{\rho (g/cm^3)}$$

$$\underset{(eV)}{U_E} = \frac{10^{-10}}{72} \times E\epsilon^2 \frac{M_w (g/mol)}{\rho (g/cc)}$$

$M_w = 123$ g/mol. $E = 200$ GPa

$e = 1\%$

$\rho = 5.68$ g/cc.

$$= \frac{10^{-10}}{10^2 \times 7.2} \times \frac{1.23 \times 10^2}{5.68} \times 2.00 \times 10^{9+2} \times 10^{-4}.$$

$$= 10^{-10-\cancel{2}+\cancel{2}+11-4} \times \frac{1.23 \times 2}{0.75 \times 5.68}$$

$\simeq 0.5\,10^{-3}$ eV (meV) 1% strain

Now → 20% strn $\frac{0.20}{0.01} = 20$

$\simeq 10^{-3} \times 20 \times 20 = \underline{0.4 eV}$

$\Delta H_s = 1080 \times$ kJ/mol. 100 kJ/mol $\simeq 1$ eV

$\simeq 10$ eV

Copper

$$U_{eV} = \frac{10^{-10}}{72} \times E\epsilon^2 \times \frac{M_w(g/m)}{\rho(g/m)}$$

$E = 128\ GPa$. $M_w = 63.5\ g/mol$.

$\rho = 8.96\ g/cm^3$

Heat of Evaporation $= 300\ kJ/mol \approx 3\,eV$

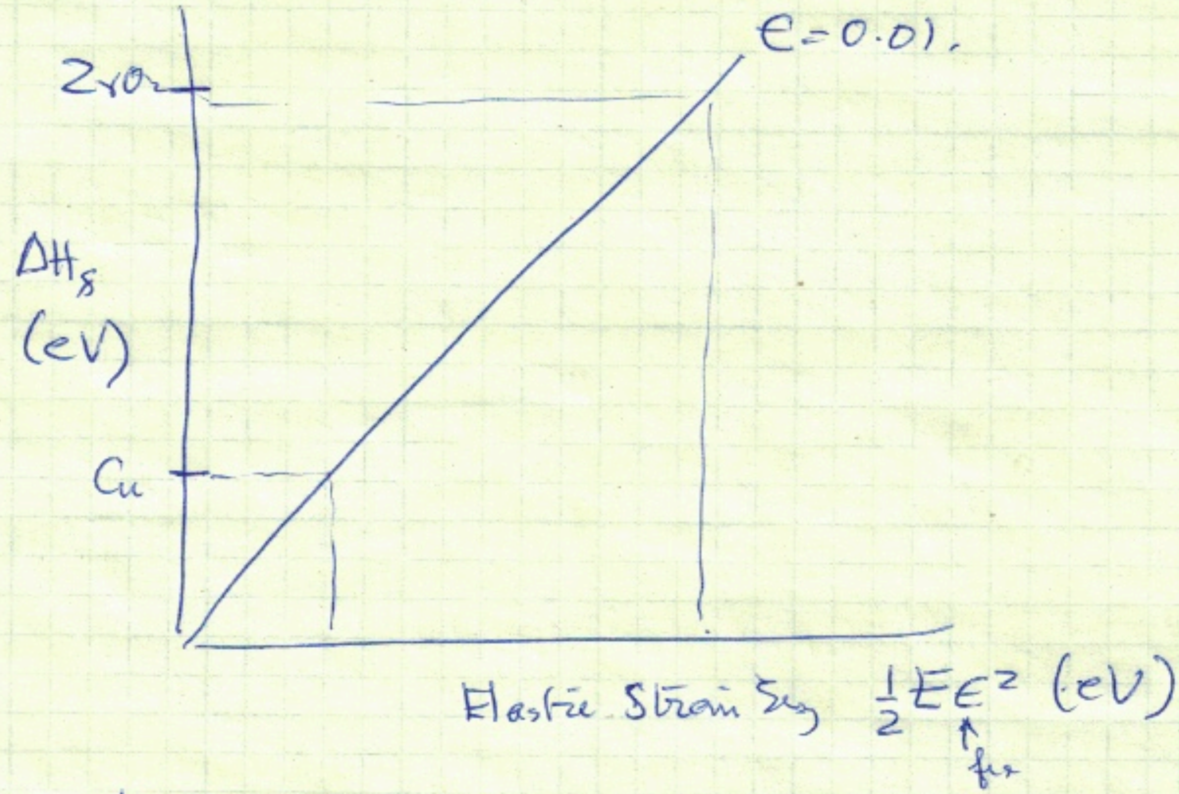
$$U_{eV}(1\%\ \text{strain}) = \frac{10^{-10}}{72} \times 128 \times 10^{9} \times 10^{-4} \times \frac{63.5}{8.96}$$

$$= \frac{63.5 \times 128}{8.96 \times 0.72} \times 10^{-10-4+9}$$

$$= 12.5 \times 10^{-5}\ eV$$

$$= \cancel{0.012\ eV}\ 0.125\ meV$$

	ZrO_2	Cu
U_{eV} (1% st)	~~0.5 × 10~~ 0.5 meV	0.125 meV
ΔH_f (eV)	10	3
Ratio $\frac{\Delta H_f}{U_{eV}}$	20	24



$$\boxed{\Delta H_f \propto E}$$

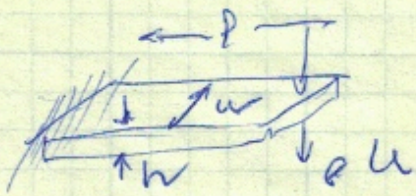
Elastic Constant

E

↑

Material Prop.
(hand book value)

(BV problem)
Mechanics
→
Continuum.



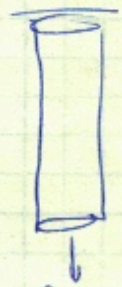
cantilever beam.

$$\frac{\Delta u}{P} = \frac{4 l^3}{w h^3 E}$$

Atomic Force Microscope

Airplane wing

Material Selecti



$$k = \frac{P}{\Delta L} = \frac{P}{A} \times \frac{1}{\frac{\Delta L}{L}} \times \frac{A}{L}$$

k = stiffness. $= \frac{\sigma}{\epsilon} \cdot \frac{A}{L}$

$$\frac{N}{m} \leftarrow k = E \times \frac{A}{L} = \frac{EW}{L^2 \rho}$$

$$W = A \times L\rho$$

$$\frac{N}{m^2} \times \frac{m^3}{m^2} \cdot \rho$$

$$k = \frac{E}{\rho} \times W \times L^2$$

Show on a graph

Cantilever Beam

Atomic Force Microscope

$$\frac{4l^3}{w h^3 E} = \frac{1}{k}$$

DNA $P = 10\ pN$

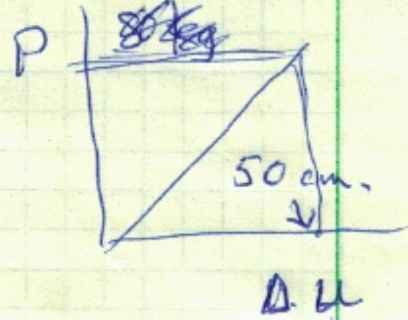
$\Delta u = 10\ \mu m$

$$k = \frac{w h^3 E}{4 l^3}$$

$$k = \frac{10 \times 10^{-12}}{10 \times 10^{-6}} = 10^{-6}\ N/m$$

Diving Board

$$k = \frac{wh^3 E}{4 l^3} \quad N/m.$$



$$U_E = \frac{k\,\Delta u^2}{2}$$

$$U_E^{DESIRE} = 80\,kg \times \quad Mgh$$

$$W(kg) \times H_{jump}$$

$$= \underbrace{W \overset{(kg)}{} \times 10}_{N} \times \underbrace{H_j}_{m}$$

$$W(kg) \times 10 \times H_j = \frac{wh^3 E}{4 l^3} \times \frac{\Delta u^2}{2}$$

$$E = (W(kg) \times 10)\, H_j \;\frac{8 l^3}{w h^3 \Delta u^2}$$

83 GPa

$$\frac{N \times m \times m^3}{m \times m^3\, m^2} = \frac{N}{m^2}$$