# Lecture Notes for Monday 03/30/2020

## Topics

(i) Review mechanical energy available to propagate a crack.

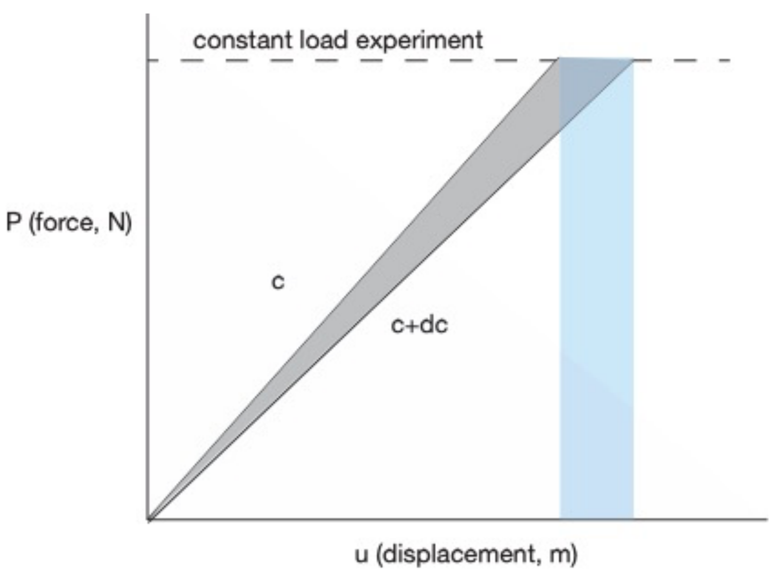
(ii) The criterion for the critical condition.

(iii) Analysis for double cantilever beam specimen

(iv) Number into (iii)

(v) Generality of the method to other specimen geometries.

## (i) Review mechanical energy available to propagate a crack.




*•Fracture is described by the surface area of the crack. To describe the fracture energy properly, need to multiply c by the thickness of the sample, if consider a two dimensional.*
*•In an experiment at constant load the specimen becomes becomes more compliant when the crack grows, so the slope for the line for (c+dc) is more compliant than for "c".*
*•If the crack grows then P does work on the system by reducing its potential energy, and this work is shown by the area in blue. But at the same time the elastic energy stored in the body will increase by the amount shown by the triangle in grey color. Therefore the total mechanical energy available to propagate the crack is one half of P*δu since the triangle has one half the area of the rectangle.*

Practice Problem: Show that the mechanical energy available in a constant displacement experiment — as done in the laboratory with a stiff screw driven Instron Machine — is equal to the area within the triangle.

The message is: The mechanical energy available for crack propagation can be written either as (a) one half of the change in potential energy, or (b) the change in the stored elastic energy. The second scenario is useful when analyzing fracture from a penny shaped crack embedded in body under a remote tensile stress σ.

## (ii) The criterion for the critical condition.

Consider the load displacement profiles drawn above. The fracture criterion is,

$$\frac{1}{2}P^*\delta u \geq 2\gamma_F \delta c w \qquad (1.1)$$

Now, we can see that $\delta u$ will be related to how much the compliance changes when the crack grows from c to c+δc.

$$\frac{dS}{dc} = \frac{d\left(\frac{u}{P}\right)}{dc} = \frac{1}{P}\frac{du}{dc} \qquad (1.2)$$

Note: P can be pulled to the front since it is constant as defined by the experiment that I am analyzing.

Rewrite Eq. (1.1)

$$2\gamma_F \geq \frac{1}{2}\frac{P^*}{w}\frac{du}{dc} \qquad (1.3)$$

Combine (1.2) and (1.3)

$$2\gamma_F \geq \frac{1}{2}\frac{P^{*2}}{w}\frac{dS}{dc} \qquad (1.4)$$

Check that units are balanced, since $\gamma_F$ has units of force per unit length, and S has units of length over force.

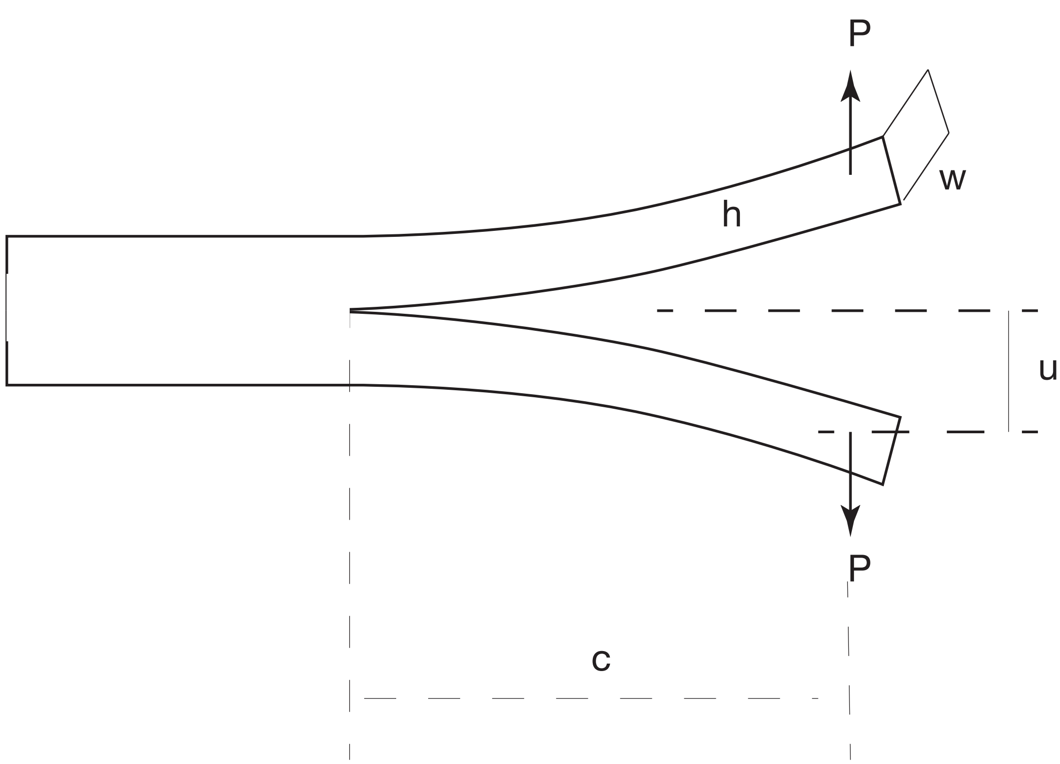
The great value of Eq. (1.4) is that (dS/dc) can be determined entirely by an experiment but cutting in cracks of different length into the sample and measure the compliance, and get the slope of the graph of one against the other. Of course if (dS/dc) can be analyzed with simple mechanics then all the better.

Would like to present (1.4) in terms of the fracture toughness, $K_{IC}$, which is the engineering parameter for fracture analysis. We have shown that

$$2\gamma_F = \frac{K_{IC}^2}{E} \qquad (1.5)$$

## (iii) Analysis for double cantilever beam specimen

This specimen is like two cantilever beams merging into a single solid, and fracture is induce by pulling apart the ends of the two beams.



From our earlier study of the cantilever beam (as in the design of a atomic force microscope) we have that

$$S = 2\frac{u}{P} = 2\frac{4c^3}{wh^3E} \quad (1.6)$$

Therefore,

$$\frac{dS}{dc} = 2\frac{12c^2}{wh^3E} \quad (1.7)$$

The fracture criterion becomes

$$2\gamma_F \geq P^{*2}\frac{12c^2}{w^2h^3E} \quad 2\gamma_F \geq \frac{1}{2}\frac{P^{*2}}{w}\frac{dS}{dc}$$

$$K_{IC}^2 = P^{*2}\frac{12c^2}{w^2h^3} \quad (1.8)$$

Units: LHS ($force^2/length^4$)*length. RHS checks.

(iv) Consider application of (1.8) to an experiment.

Fracture in a glass slide (silica). The slide is 10 cm long, 2 cm wide, with thickness of 2 mm. Cut a crack from one edge half way along the length of the slide,

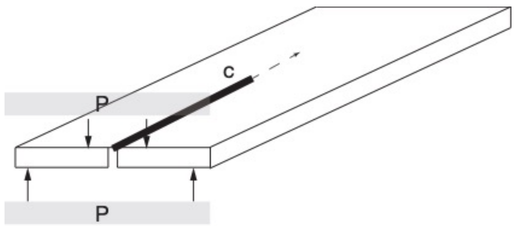
C = 5 cm

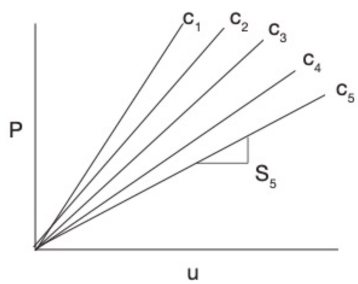
H = 1cm

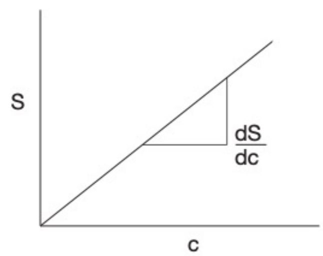
W = 0.2 cm

E (glass) is 73GPa

## (v) Generality of the method to other specimen geometries




Method

•Make five specimens with different lengths of “c” cut into them.

•Measure the compliance for each of them

•Make a plot of compliance against “c” to obtain (dS/dc)

Now load the crack to fracture to determine $P^*$ for a given crack length. Substitute in

$$2\gamma_F \geq \frac{P^{*2}}{2}\frac{dS}{dc}$$

, to obtain the fracture energy, and then substitute I in Eq. (1.5) to obtain $K_{IC}$. Note the “w” is not present in the equation as it is in Eq. (1.4) because this is a three dimensional problem.