Lecture Notes from Monday 04/13/202
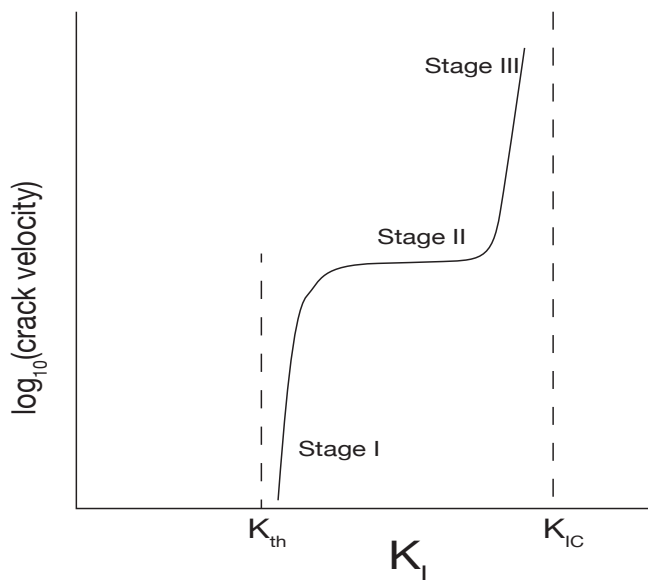
Topic: Sub-Critical, also called Slow Crack Growth (because it is time dependent)

Overview

If a crack in silica glass is loaded with a stress-intensity factor, K_I such that

$$K_I < K_{Ic} \quad (2.9)$$

and the specimen is immersed in water which is corrosive to glass, then the crack grows slowly with time. When these data are plotted as the crack velocity versus K_I the following type of behavior is seen:

Note that usually the logarithm of the crack velocity is plotted against K_I (sometimes the stress intensity is also plotted on a log scale).

The crack growth behavior is characterized by three stages:

Stage I: a threshold like behavior where the crack velocity tends (asymptotically) to near zero. This part of the curve is described by a threshold stress intensity, K_{th}, which can be used in engineering design since below it the crack essentially does not grow.

Stage II: Stable crack growth rate which is essentially insensitive to K_I.

Stage III: where the crack grows quickly like in classical brittle fracture when the stress intensity factor approaches K_{Ic}.

The figure on the following page is from a classical paper published in 1965 (it is still cited since it was the original work — something in keep in mind about the important of being truly original). You can see the original paper at this link:
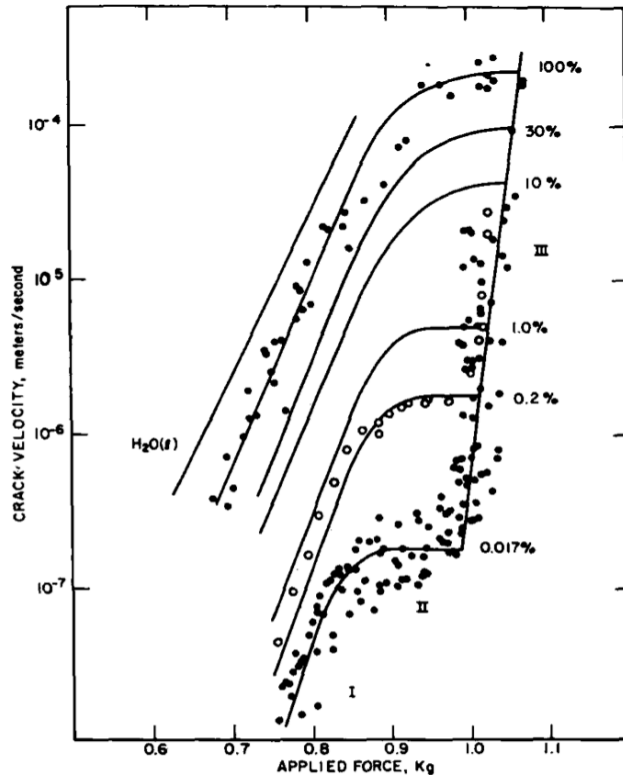
Fig. 3. Dependence of crack velocity on applied force. The percent relative humidity for each set of runs is given on the right-hand side of the diagram. Roman numerals identify the different regions of crack propagation. Region III shows only about half the points used to determine the line. The line, therefore, does not seem to fit the data as well as it would if all data in region III were plotted.

The graphs show the influence of humidity on the rate of crack growth, from near zero humidity to 100% humidity. All curves merge when approaching K_{IC}.

The K_{IC} for glass is~1MPa $m^{1/2}$, so slow crack growth occurs below this value, from 0.7–0.9 MPa $m^{1/2}$.
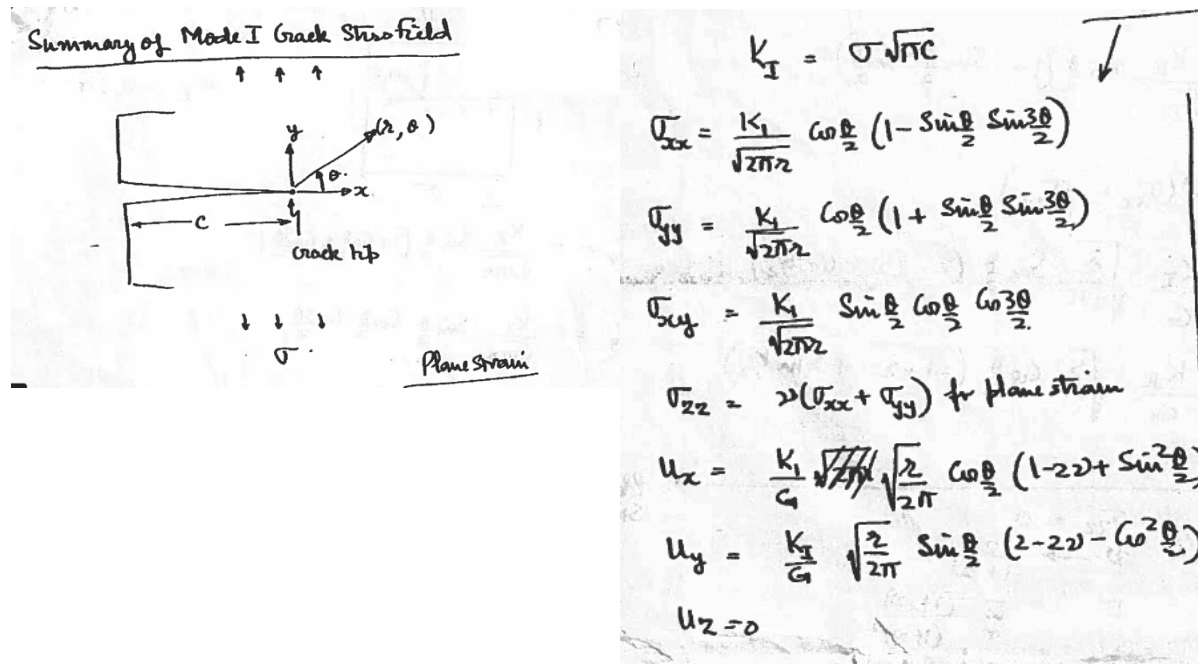
Note that the crack velocity, plotted here on a logarithmic scale in units of m sec^{-1}, ranges from 10^{-8} to 10^{-1} m s^{-2}.

Mechanisms

•Stage I: The "threshold", that is, the Stage I behavior is explained by the corrosion molecule, e.g. water, being too large to be able to access the crack tip.

•Stage II: The crack velocity becomes constant (at a given level of humidity) since the rate of corrosion is related to the rate of transport (we call it diffusion) from the environment to the crack tip. The mechanism of diffusion is at the surface (surface diffusion), therefore it is independent of K_I, and therefore, independent of the CTOD (since CTOD) increases with the stress intensity.

•Stage III: this is related to the intrinsic fracture resistance (fracture toughness) of the material, and therefore does not depend on the atmospheric humidity.

Quantitative Analysis

The mechanism described above can be quantitatively analyzed from the results of stress analysis at the crack tip. Note that the stresses and the displacements around the crack tip, as given below, are described in terms of the stress intensity factor, since this is the loading parameter for the crack.

Summary of Mode I Crack Stress field

$$K_I = \sigma\sqrt{\pi c}$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \text{ for plane strain}$$

$$u_x = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(1 - 2\nu + \sin^2\frac{\theta}{2}\right)$$

$$u_y = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(2 - 2\nu - \cos^2\frac{\theta}{2}\right)$$

$$u_z = 0$$

(Plane Strain)

What concerns in the present problem is the opening of the crack in front of the crack tip at a distance of about the intermolecular spacing. Please convince yourself that this value of CTOD$_{th}$ with the "th" subscript specifying that we seek to explain the threshold behavior in the figure during Stage I, is given by

$$\text{CTOD}_{th} = 2u_y(r = 0.5nm, \theta = 180^o) \quad \text{Eq. (2.10)}$$

Note that: (i) r is equal to be about equal to intermolecular spacing in glass which is assumed to be 0.5 nm, and $\theta = 180^o$ so that the location is in front of the crack tip.

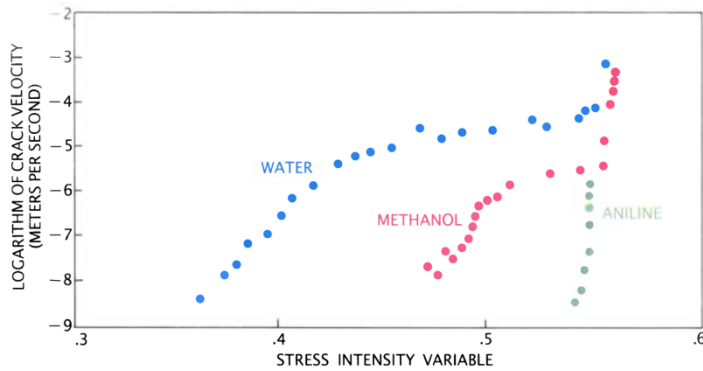The condition for threshold is then obtained by the following condition

$$M \geq CTOD_{th}$$

Eq. (2.11)

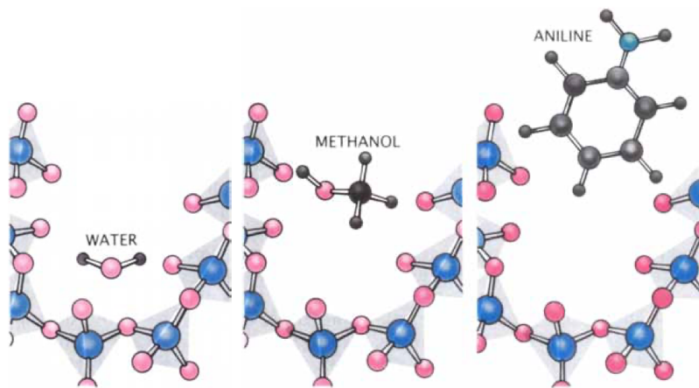where M is the physical size of the molecule.

Application of Eq. (2.11)

Consider the data from Michalski and Bunker drawn from the following reference:

On the left note how the threshold stress changes with the size of the molecules, as illustrated by the schematic in the bottom figure. The larger the molecule the larger is the value of K_{th}.

The molecular diameters of several molecules is given in the following table:

MOLECULAR SIZE of a chemical substance affects its ability to speed the growth of cracks in glass. Water, which has a molecular size of only .26 nanometer (billionth of a meter), causes cracks to grow much faster than methanol (.36 nanometer) does, and aniline (.42 nanometer) has hardly any effect (*top*). In essence, water can readily enter a crack opening (which has a diameter of .4 to .5 nanometer), methanol has difficulty getting in and aniline is so big that it never arrives at the sites of bond rupture (*bottom*).

Table II. Molecular Diameters of Test Chemicals

Chemical	σ (nm)
Water	0.265
Methanol	0.359
Ammonia	0.260
Hydrazine	0.36
Formamide	0.40
N-Methylformamide	0.45
n-Propylamine	0.46
tert-Butylamine	0.52
Aniline	0.55

The table above is drawn from
https://ceramics.onlinelibrary.wiley.com/doi/epdf/10.1111/j.1151-2916.1987.tb04879.x

Practice Homework

Apply the procedure explained in Eq. (2.10) and the data in the above figure and table, to check if it applies to the data for the threshold values of the stress intensity factor for water, methanol and aniline.