Spring 2020: Mechanical Properties Materials Science (MPMS)

Take Home Exam III: (Fracture)
Given out on Friday 04/13/2020, due on Monday 04/20/2020

Each question carries equal points.

1. Give three iconic examples, in the form of a figure and/or an equation, along with a couple of sentences to explain, of phenomena related to "fracture".

(i) $K_{IC} = \alpha\sigma\sqrt{c}$

The fracture toughness is a material property that prescribes the relationship between flaw size and the fracture stress. α is a parameter that depends on the geometry of the flaw.

(ii) Fracture strength of glass fibers increases as the fiber diameter decreases.

(iii) The ideal fracture strength of solids, that is in the absence of physical flaws, is approximately 10% to 15% of the Youngs modulus.

2. Please see notes where the sinusoidal form of the force-displacement curve for bond stretching leads to the following equation

$$2\gamma_F = \frac{E\Omega^{1/3}}{2\pi^2} \qquad (1)$$

In the past we have used a similar approach to derive a relationship between the modulus, E, and the enthalpy of formation ΔH_V Combine these two analyses to obtain a relationship between $2\gamma_F$ and ΔH_V.

Note: in the sinusoidal curve the initial slope is related to E, and the area within the curve to the work of fracture.

From notes on Elasticity we have that,

$$E = \frac{8\pi^2}{3\Omega}\Delta H_V \qquad (2)$$

Combining (1) and (2) we obtain

$$2\gamma_F = \frac{4\Delta H_V}{3\Omega^{2/3}}$$

Note that ΔH_V has units of J per atom, while $\Omega^{2/3}$ has units of area per atom. Thus $2\gamma_F$ has units of J m^{-2}.

3. Design a double cantilever fracture experiment to measure the fracture toughness of wood. You may assume the length of each cantilever to be 25 cm, the vertical height to be 7.5 cm, and the thickness to be 2.5 cm. (think of a 1x6 plank of wood).

Read the value of the fracture toughness from the notes and calculate the load that would cause the beam to fracture.

From notes you have that

$$K_{IC}^2 = 12P^{*2}\frac{c^2}{w^2h^3}$$

where,
K_{IC} = 1 MPa $m^{1/2}$ (from the fracture toughness map)
C = 25 cm (length of (each) cantilever beam
w = 2.5 cm the thickness of the beam
h = 7.5 cm the height of each beam.

Substituting above gives

P^* = (approx..) 600 N, i.e. ~60 kg or 120 lbs (somewhat less that the weight of an average person)

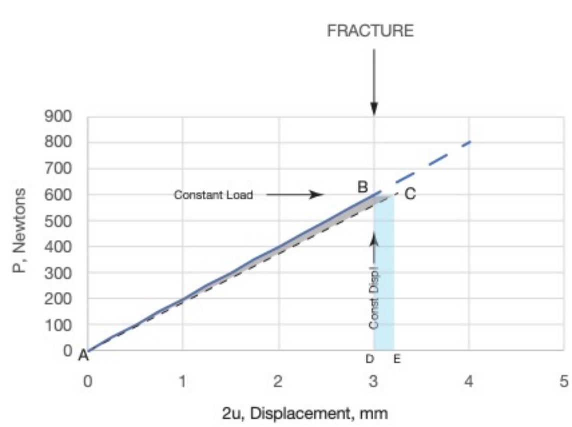
Note that by increasing h to 10 c, will increase the load bearing capacity to 180 lbs

4. The experiment in Problem 3 was a constant load experiment. Now do the same experiment by controlling the displacement by driving a wedge between the two arms of the double cantilever beam specimen. Draw a load vs displacement curve for this scenario and mark the point of fracture on this graph. Draw the distinction between this constant displacement experiment and the constant load experiment in Problem 3 (in a few words).

The displacement (remember the factor of 2 to account for displacement in both wings of the double cantilever beam) at applied load P is given by (see notes online)

$$2u = \frac{8c^3}{wh^3E}P$$ (note that the units are consistent)

The Young's Modulus, E, is approximately 1 GPa. A plot of the above Equation using the values given in Prob 3 is plotted below. The range is for P up to 800 N since we know that fracture occurs at about 600 N.



In a constant load experiment the displacement increases from B to C; therefore the potential energy decreases by the area of the blue triangle. At the same time the stored energy increases from ABD to ACE, the difference being the triangle filled in grey, which, in magnitude (but opposite in sign) is one half the area of the rectangle.

In a constant displacement test, the cantilever is drawn apart with a wedge to control the displacement and fracture occurs when the load at B is reached. The stored energy now decreases by the area within the grey triangle. There is not change in the potential energy since the displacement is constant.

5. Please refer to the map where the Fracture Toughness is plotted against the Young's Modulus. What are the values for GFRP (graphite fiber reinforced polymers) and simple polymers such as epoxy?

Explain the large difference between the fracture toughness of these two materials although even though the matrix phase in GFRP is the epoxy.

The fracture toughness of exoxy is about 1 GPa, while it is ~80 GPa for GFRP. Both fracture by the same fracture mechanism, i.e. crazing in the polymer. The work of fracture for both materials is the same. However, recall that

$$2\gamma_F = \frac{K_{IC}^2}{E}$$

The left hand side is the same for the epoxy and the GFRP. However, since E for GFRP is greater by a factor of about 100, GFRP has ten times the fracture toughness of epoxy.

6. Apply Eqns (2.7) and (2.8) to calculate the CTOD and the plastic zone size, Z (at the point of fracture) for the following materials,
-silica glass
-polymer, e.g. polypropylene or PMMA
-silicon carbide
-titanium
-GFRP
The data for KIC as a function of E, and the data for the yield strength, Y or *σ#*, as a function of E are given on the Fracture Page of the website, and also on the following next two pages.

$CTOD = \frac{K_{IC}^2}{YE}$, and the plastic zone size is given by $Z = \frac{K_{IC}^2}{Y^2}$

	E	Y	K_IC	CTOD	Z
	Pa	Pa	Pa m^1/2	um	um
Silica	8.00E+10	1.00E+08	1.00E+06	1.25E-01	1.00E+02
Epoxy	8.00E+09	8.00E+07	1.00E+06	1.56E+00	1.56E+02
SiC	3.50E+11	8.00E+08	5.00E+06	8.93E-02	3.91E+01
Ti	1.00E+11	7.00E+08	1.00E+08	1.43E+02	2.04E+04
GFRP	8.00E+10	1.00E+09	3.00E+07	1.13E+01	9.00E+02

Note that (i) the CTOD values range from ~0.1μm (lowest for glass) to 143 μm for titanium, (ii) the plastic zone size reaches about 2 cm for titanium. Therefore the plastic zones can be rather easily measured in optical microscopy as in the Hahn and Rosenfield paper.

Whether the specimen fractures by crack propagation or by general yielding (called ductile fracture) depends on the size of the specimen relative to the plastic zone size.

Therefore while fracture in titanium is likely to occur as “ductile fracture” (because the plastic zone size is likely to be similar to the size of the specimens) while the other materials with a plastic zone size of well under 1 mm will fail in a brittle or semi-brittle manner.