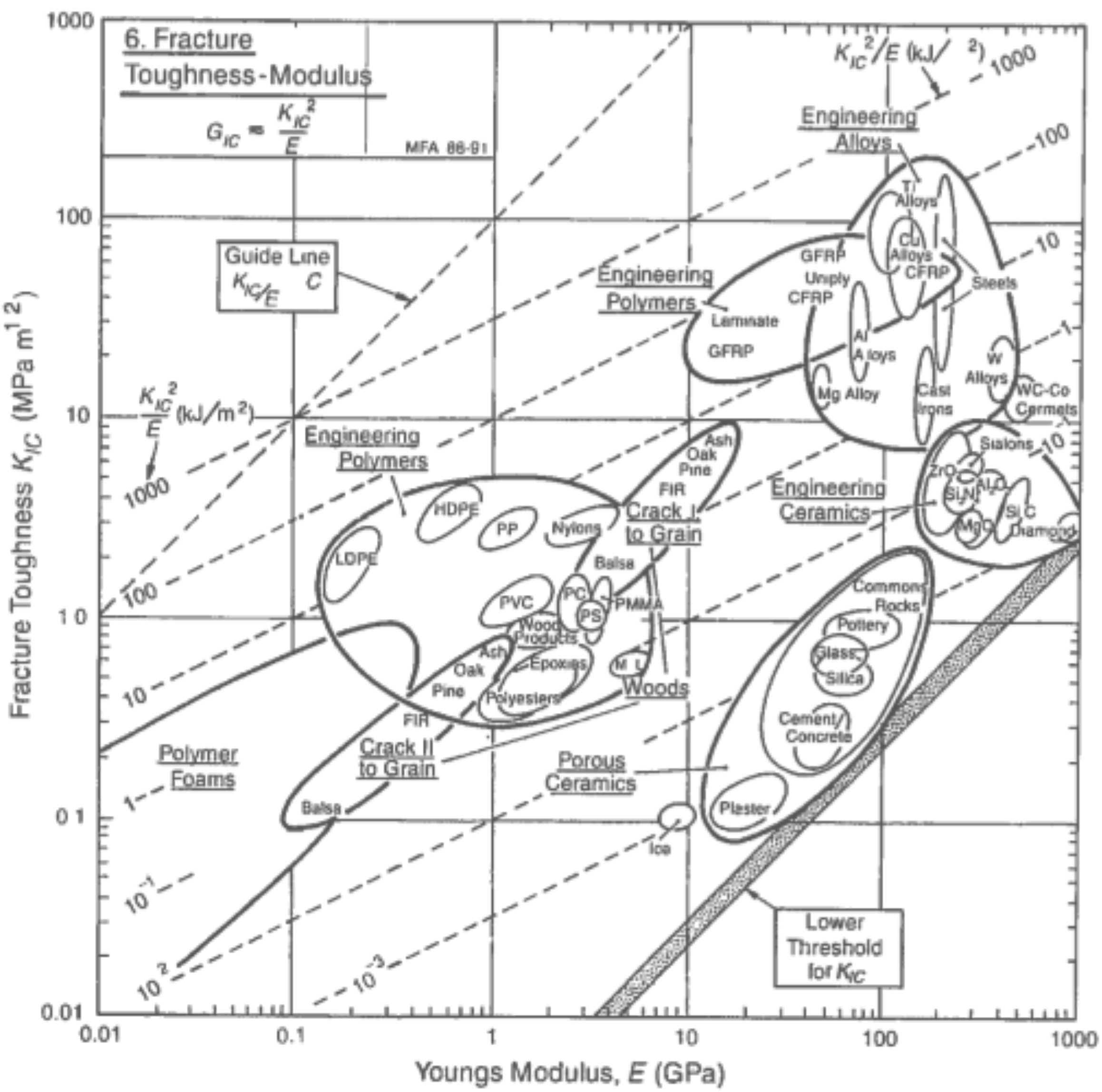
2C_Fracture Toughness

Definition and Units

Fracture toughness is written as

K_{IC} which has units of $MPa\sqrt{m}$ (Mega Pascals root meters); note that

The subscript "c" represents the critical values (the fracture limit) of K_I which is called the stress intensity factor. It combines the applied stress (MPa) with the flaw size (m). K_I, which is called the stress-intensity factor, represents local stress field and the displacement-field at the crack tip. Since it is the stress concentration at the crack tip which can precipitate fracture (that is, propagation of the crack), the fracture property of the material is described in terms of K_I; fracture occurs when $K_I \geq K_{IC}$, where K_{IC} is a "handbook" property of the material, which spells its resistance to crack propagation: hence it is called the fracture toughness of the material.



The fracture toughness for many classes of materials are shown in the map where K_{IC} is plotted against the Youngs Modulus. Once again the Youngs Modulus becomes a guide to fracture resistance. Typical range of values for different materials are:

Glasses (silica for example)	1 to 2 MPa $m^{1/2}$
Engineering Ceramics (zirconia, silicon carbide, alumina)	5 to 10 MPa $m^{1/2}$
Engineering Metals (tungsten carbide, titanium, etc.)	10 to 100 MPa $m^{1/2}$

The stress-intensity factor combines the applied stress and the flaw size in the following manner

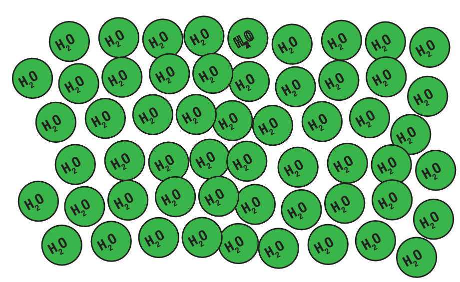
$$K_{IC} = \sigma_F \sqrt{\alpha c} \ \text{MPa m}^{1/2} \qquad (1)$$

Here α is a dimensionless parameter which depends on the shape of the flaw. It is of order unity. Typically, it has a value ranging from unity to π.

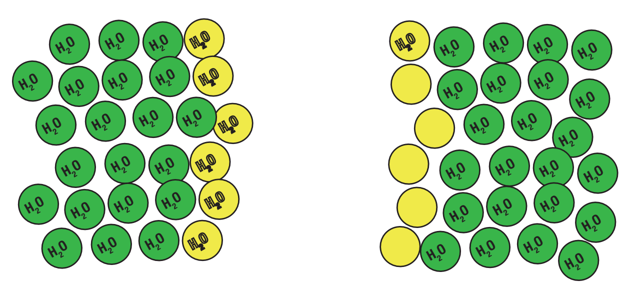
The Significance of Surface Energy

Fracture of a solid is accompanied by the creation of new surface. For example consider the fracture of ice. Fracture creates two surfaces. Therefore we can envision that cracks propagate by successive scission of bonds at the crack tip. As the bonds break, two fresh surfaces are created, as shown by the yellow atoms on the right. The work done to break the bonds is related to the "work of fracture". This work of fracture is given units of J m^{-2}, that is, the work done to extend the crack surface by a square meter. It is written as:

State I



State II



$2\gamma_F$ in J m^{-2}

It is equal to the work of breaking N_S bonds where

$N_S = \frac{1}{\Omega^{2/3}}$, that is the number of atoms per unit area of surface.

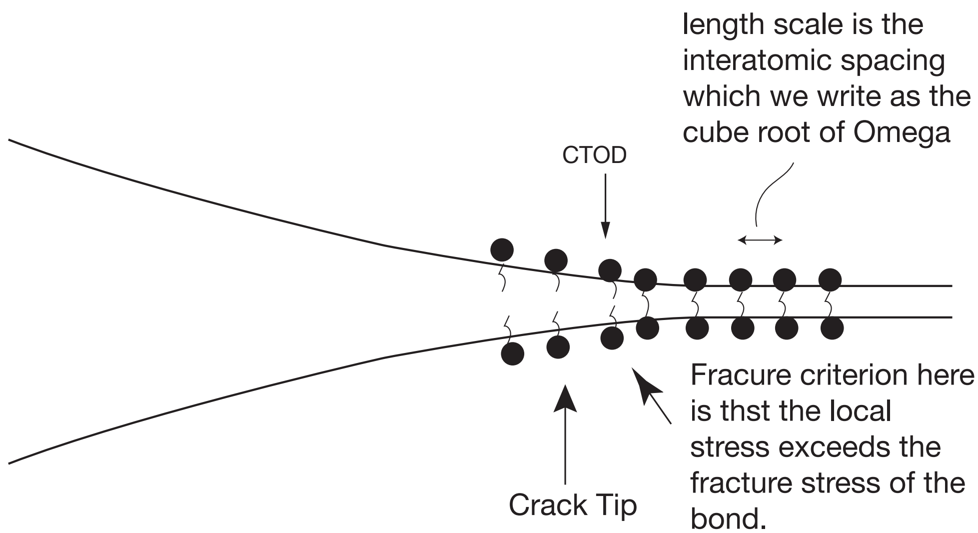
It can be shown that

$$2\gamma_F = \frac{K_{IC}^2}{E} \qquad (2)$$

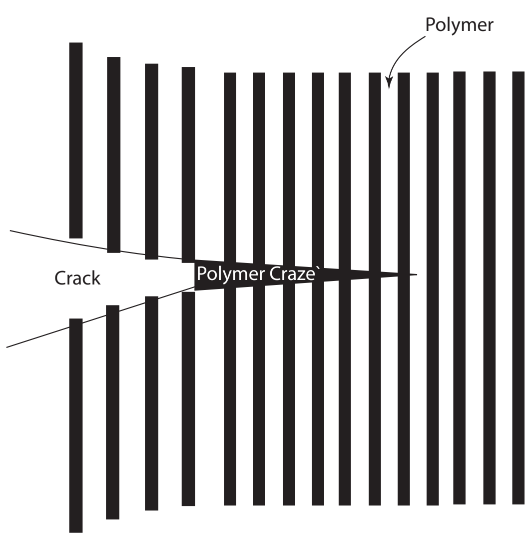
where E is the Youngs modulus. Note that the data shown in the map above follows that guideline predicted by Eq. (2) that the fracture toughness increases with the Youngs Modulus.

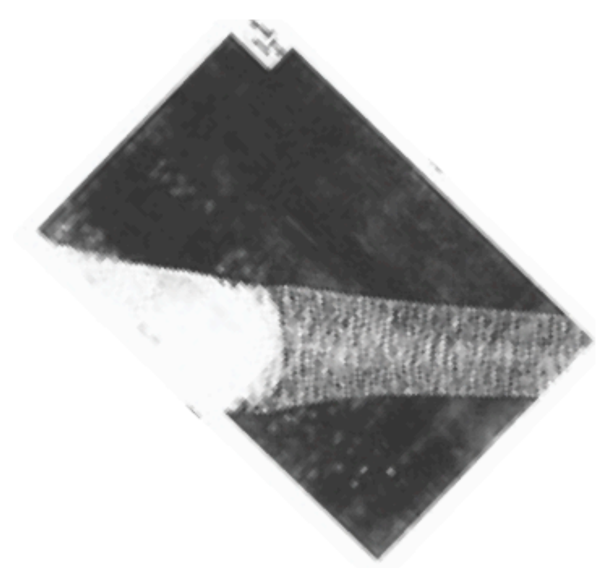
Mechanism of Fracture at the Crack Tip

The mechanism of fracture is related to the events at the crack tip that lead to propagation of the crack. The simplest scenario is where individual bond break one at a time to propagate te crack as shown in the figure with green atoms within the solid and yellow atoms on the free surface. Translated into the propagation of a crack tip,



But there may be other mechanisms as well which require much more work to break the bonds. For example, in polymers the bonds are the crack tip stretch until they break. This stretching of the bonds at the crack tip is shown below





The work done to break a bond, is the integral of force times the displacement. In polymers this integral is large because considerable displacement is need to break the fibrils. Therefore,

$$\left(2\gamma_F\right)_{polymers} \gg \left(2\gamma_F\right)_{brittle\,glass}$$

Yet we see in the map that both the polymer and the glass have similar values for K_{IC}. The reason why this is so is because

$$\left(E\right)_{polymers} \ll \left(E\right)_{brittle\,glass}$$

From Eq. (2) we have that:

$$K_{IC}^2 = 2\gamma_F E \qquad (3)$$

Therefore, which the work of fracture is higher in polymers, their low elastic modulus reduces their fracture toughness.

Remember that the fracture toughness $K_{IC} = \sigma_F \sqrt{c}$ is the engineering design parameter since is includes both the fracture stress and the flaw size.

Further Comments on Eq. (2)

$$2\gamma_F = \frac{K_{IC}^2}{E} \qquad (2)$$

The equation related three "hand book" parameters for a material. Among them $2\gamma_F$ is the work of fracture that is the work done to break bonds multiplied by the number of bonds per unit area, E is the elastic modulus which is related to the elastic stiffness of the bonds., while K_{IC} is the fracture toughness which couples the fracture stress to the flaw size. Rewriting Eq. (2):

$$K_{IC}^2 = 2\gamma_F E \qquad (3)$$

Therefore, even if the work of fracture is high, a lower modulus can lower the effective fracture toughness of the material.

engineering resistance of the material to fracture.

For example in the map given above note that the fracture toughness of glass and polymers is nearly the same (about 1 MPa $m^{1/2}$) even though as evident from the two sketches on the previous page, the work to propagate the crack in polymers, where the fibrils have to be stretched significantly to advance the crack, is much larger than the work done to break bonds at the crack tip in a brittle material such as glass.

Indeed the data in the map allows us to estimate how much greater $(2\gamma_F)_{polymers}$ is relative to $(2\gamma_F)_{glass}$. Since both materials have approximately the same value of $K_{IC} = 1\ MPa\sqrt{m}$, and because $(E_{glass} / E_{polymers}) \approx 100$, we estimate that

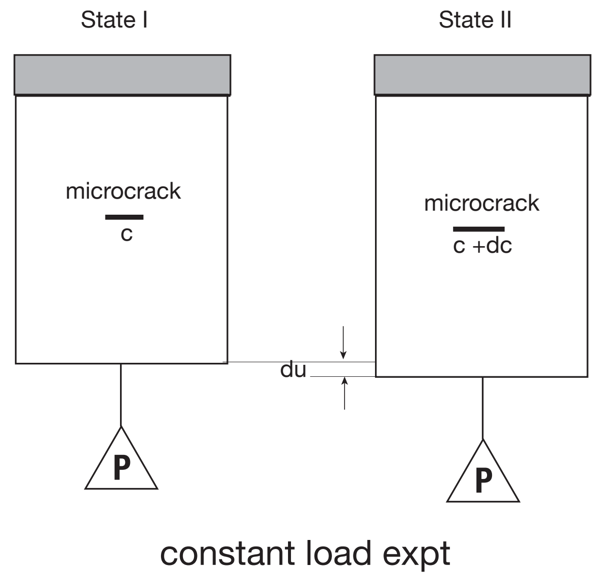
$$\frac{(2\gamma_F)_{polymers}}{(2\gamma_F)_{glass}} \approx 100$$

Next we shall try to understand why the elastic modulus plays a role in the fracture resistance of a material.

The Role of the Elastic Modulus

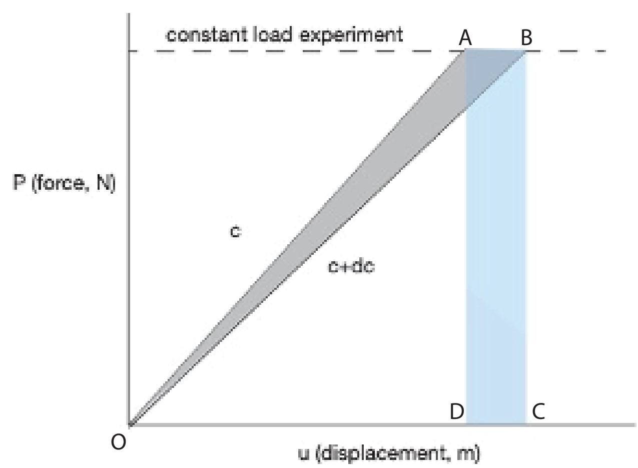
For the crack to propagate the mechanical work expended to propagate the crack must be greater than the work of fracture.

The change in the mechanical work when the crack grows from $c \rightarrow (c + \delta c)$ has two parts: (i) the work done on or by the surroundings on the system, and (ii) the change in the stored elastic energy in the system. We consider these two terms in the following way,



The sketch on the right shows the crack of size c, in State I, grow into $c + dc$ in State II. The potential energy of the system decreases by $P.du$ which is equal to the area of the blue rectangle shown below, as ABCD. This work can be used to grow the crack.

However, the stored elastic energy in the system also changes. The stored energy changes from the initial area of the triangle, OAD to the triangle OBC.



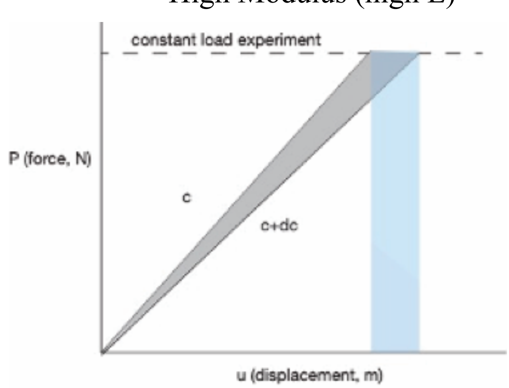
Note that OBC > OAD by the sliver of the triangle OAB. Moreover in magnitude Area (OAB) = (1/2)Area (ABCD). Noting that while OAB increases the (stored mechanical energy), ABCD decreases the total potential energy **Therefore the total mechanical energy of the system is *<u>decreased</u>* by**

$$\frac{1}{2} \text{ Area (ABCD)} \qquad (4)$$

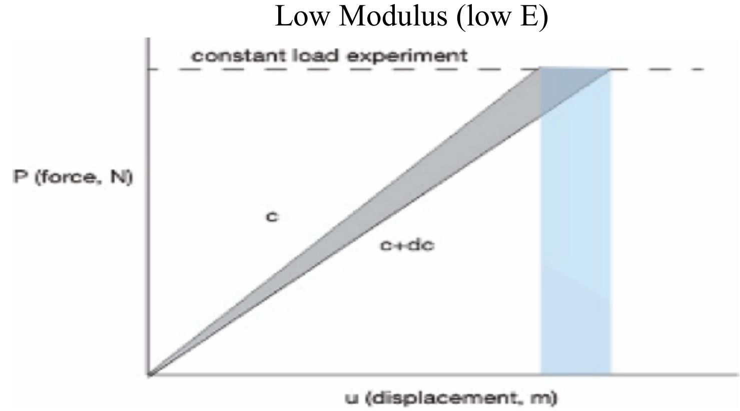
This is the mechanical energy available to overcome the work of fracture.

I will leave it to you to study what the quantity given by Eq. (4) will increase in magnitude (that is increase the work available to overcome the work of fracture) for a lower modulus material. For example consider the two figures below, the one on the right having a lower modulus, that is a higher compliance.

High Modulus (high E)



Low Modulus (low E)



It is easy to read that the work available for fracture is greater for the low modulus material (since the area of the rectangle is greater) for the lower modulus material.